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Superstring 'ending' on super-D9-brane: a supersymmetric action functional for the coupled brane system.

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Abstract

A supersymmetric action functional describing the interaction of the fundamental superstring with the $D = 10$, type IIB Dirichlet super-9-brane is presented. A set of supersymmetric equations for the coupled system is obtained from the action principle. It is found that the interaction of the string endpoints with the super–D9–brane gauge field requires some restrictions for the image of the gauge field strength. When those restrictions are not imposed, the equations imply the absence of the endpoints, and the equations coincide either with the ones of the free super-D9-brane or with the ones for the free closed type IIB superstring. Different phases of the coupled system are described. A generalization to an arbitrary system of intersecting branes is discussed.

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1 Introduction

Intersecting branes and branes ending on branes receive much attention now [1]–[9] in relation with the development of M-theory [10] and its application to gauge theories [11, 12]. However, the studies of [1]–[9] were performed for the pure bosonic limit of the brane systems or for a supersymmetric description in the framework of the 'probe brane' approach only.

In the first case they are based upon the observation that the ground state should not include the nontrivial expectation values of the fermions in order to keep (part of) the Lorentz invariance (corresponding to the configuration of the branes). Then it is possible to justify that some of the pure bosonic solutions preserve part of the target space supersymmetry and, just due to this property, saturate the Bogomolnyi bound and are thus stable (see e.g. [13] and refs. therein).

In the second case one of the branes is treated in an the 'external field' of the other brane. The latter can be considered either as the solution of low energy supergravity theory [1, 3], or, in the frame of the superembedding approach [14, 15, 16, 17, 18], as a superspace, where the ends of the probe brane are living [6, 7, 8]. In such an approach the κ-symmetry of the probe brane plays an essential role for studying the 'host' brane and the coupled system.

Despite many successes of these approaches, it is desirable to obtain a complete and manifestly supersymmetric description of interacting brane systems.

Of course, the preservation of supersymmetry in the presence of boundaries (including the boundaries of open branes ending on other branes) requires the analysis of anomalies [19, 2], while at the classical level the boundary breaks at least half of the supersymmetry [20, 21, 2]. So at that level one may search for an action for a coupled brane system, which includes manifestly supersymmetric bulk terms for all the branes and allows direct variations. The term 'supersymmetric' will be used below for an action of this type.

The main problem to be faced in a search for such an action is that the coordinate functions of intersecting branes (or of the open brane and host brane), which define embeddings of their worldvolumes, say

$$\mathcal{M}^{1+p} = (\zeta^m), \quad m = 0, \dots p \qquad and \qquad \mathcal{M}^{1+p'} = (\xi^{m'}), \quad m' = 0, \dots p'$$

into the tangent superspace $\mathcal{M}^{(D \mid N \cdot 2^{[D/2]})} = (X^{\underline{m}}, \Theta^{\underline{\mu} I})$, $\quad (\underline{m} = 0, \dots D-1, \quad \underline{\mu} = 1, \dots 2^{[D/2]}$, $I = 1, \dots N)$:

$$\mathcal{M}^{1+p} \in \mathcal{M}^{(D \mid N \cdot 2^{[D/2]})} : \qquad X^{\underline{m}} = \tilde{X}^{\underline{m}}(\zeta), \qquad \Theta^{\underline{\mu} I} = \tilde{\Theta}^{\underline{\mu} I}(\zeta)$$

and

$$\mathcal{M}^{1+p'} \in \mathcal{M}^{(1+(D-1) | N \cdot 2^{[D/2]})} : \qquad X^{\underline{m}} = \hat{X}^{\underline{m}}(\xi), \qquad \Theta^{\underline{\mu} I} = \hat{\Theta}^{\underline{\mu} I}(\xi)$$

should be identified at the intersection $\mathcal{M}^{\cap} \equiv \mathcal{M}^{1+p} \cap \mathcal{M}^{1+p'} = (\tau^r)$, $\; r = 0, \dots, dim(\mathcal{M}^{\cap}) - 1$:

$$\mathcal{M}^{\cap} \equiv \mathcal{M}^{1+p} \cap \mathcal{M}^{1+p'} \in \mathcal{M}^{(D \mid N \cdot 2^{[D/2]})} : \quad \tilde{X}^{\underline{m}}(\zeta(\tau)) = \hat{X}^{\underline{m}}(\xi(\tau)), \quad \tilde{\Theta}^{\underline{\mu} I}(\zeta(\tau)) = \hat{\Theta}^{\underline{\mu} I}(\xi(\tau)). \tag{1.1}$$

Hence the variations $\delta \tilde{X}(\zeta), \delta \tilde{\Theta}(\zeta)$ and $\delta \hat{X}(\xi), \delta \hat{\Theta}(\xi)$ may not be considered as completely independent.

Recently we proposed two procedures to solve this problem and to obtain a supersymmetric action for an interacting brane system [22]. One of them provides the necessary identification (1.1) by the Lagrange multiplier method (SSPE or superspace embedded action [22]). Another ('(D-1)-brane dominance' approach or Goldstone fermion embedded (GFE) action) involves a (dynamical or auxiliary) space-time filling brane and uses the identification of all the Grassmann coordinate fields of lower dimensional branes $\hat{\Theta}(\xi)$, $\tilde{\Theta}(\zeta)$ with the images of the (D-1)-brane Grassmann coordinate fields

$$\hat{\Theta}(\xi) = \Theta(x(\xi)), \qquad \tilde{\Theta}(\zeta) = \Theta(x(\zeta)). \tag{1.2}$$

We considered the general properties of the equations of motion which follows from such actions using the example of a superstring ending on a super–D3–brane. It was found that both approaches are equivalent and thus justify one the other. The super–9–brane was considered as an auxiliary object in [22]. The study of supersymmetric equations of motion for this system will be the subject of forthcoming paper [23].

Here we elaborate another example of the dynamical system consisting of the fundamental superstring ending on the super-D9–brane. We present explicitly the action for the coupled system and obtain equations of motion by its direct variation.

As the super-D9–brane is the space time–filling brane of the type IIB superspace, the GFE approach is most natural in this case. Moreover, the system involving the dynamical space time–filling brane has some peculiar properties which are worth studying (see e.g. [24]). On the other hand, it can be regarded as a relatively simple counterpart of the supersymmetric dynamical system including superbranes and supergravity (see [22] for some preliminary considerations).

Several problems arise when one tries to find the action for a coupled system of the space–time filling superbrane and another super–p–brane. The main one is how to formulate the supersymmetric generalization of the current (or, more precisely, dual current form) distributions $J_{D-(p+1)}$ with support localized on the brane worldvolume $\mathcal{M}^{1+p}$. Such distributions can be used to present the action of a lower dimensional brane as an integral over the D–dimensional space–time, or, equivalently, to the (D-1)–brane worldvolume. Then the action for the coupled system of the lower dimensional branes and the space-time filling brane acquires the form of an integral over the D–dimensional manifold and permits direct variation.

The solution of this problem was presented in [22] and will be elaborated here in detail. For the space-time filling brane the world volume spans the whole bosonic part of the target superspace. As a consequence, it produces a nonlinear realization of the target space supersymmetry. The expression for the supersymmetry transformations of the bosonic current form distributions (which was used in [25] for the description of the interacting bosonic M–branes [26, 27, 28]) vanishes when the above mentioned identification of the Grassmann coordinates of the lower dimensional brane with the image of the Grassmann coordinate field of the space-time filling brane is imposed. This observation provides us with the necessary current distribution form and is the key point of our construction[1].

[1]It is convenient to first adapt the description of the currents to the language of dual current forms, whose usefulness had been pointed out in [29, 30].

The second problem is related to the fact that the distributions J_{D-p-1} can be used to lift the $(p+1)$ dimensional integral to the D–dimensional one,

$$\int_{\mathcal{M}^{1+p}} \hat{\mathcal{L}}_{p+1} = \int_{\mathcal{M}^D} J_{D-p-1} \wedge \mathcal{L}_{p+1}, \qquad (1.3)$$

only when the integrated $(p+1)$-form $\hat{\mathcal{L}}_{p+1}$ can be considered as a pull–back of a D-dimensional $(p+1)$-form $\mathcal{L}_{p+1}$ living on $\mathcal{M}^D$ onto the $(p+1)$–dimensional surface $\mathcal{M}^{1+p} \in \mathcal{M}^D$.

Thus, e.g. the superstring Wess-Zumino form can be easily 'lifted' up to (i.e. rewritten as) the integral over the whole D9-brane worldvolume. However, not the entire superstring actions are written as an integral of a pull-back of a 10-dimensional form. For example, the kinetic term of the Polyakov formulation of the (super)string action

$$\int_{\mathcal{M}^{1+1}} \mathcal{L}_2^{Polyakov} = \int d^2\xi \sqrt{-g} g^{\mu\nu} \hat{\Pi}_{\mu}^{\underline{m}} \hat{\Pi}_{\nu}^{\underline{n}} \eta_{\underline{mn}} \equiv \int_{\mathcal{M}^{1+1}} \hat{\Pi}^{\underline{m}} \wedge * \Pi^{\underline{n}} \eta_{\underline{mn}} \qquad (1.4)$$

with

$$\hat{\Pi}^{\underline{m}} \equiv d\hat{X}^{\underline{m}}(\xi) - id\hat{\Theta}^I \Gamma^{\underline{m}} \hat{\Theta}^I = d\xi^\mu \hat{\Pi}_{\mu}^{\underline{m}}, \qquad \mu = 1,2 \qquad \xi^\mu = (\tau, \sigma),$$

$$\eta_{\underline{mn}} = diag(+1, -1, \ldots, -1)$$

does not possess such a formulation. Moreover, it is unclear how to define a straightforward extension of the 2–form $\mathcal{L}_2^{Polyakov}$ to the whole 10-dimensional space-time.

The same problem exists for the Dirac-Nambu-Goto and Dirac-Born-Infeld kinetic terms of super-Dp-branes and usual super-p-branes.

Though it is possible to treat the 'lifting' relation (1.3) formally (see e.g. [25] for a description of bosonic M-branes), to address the delicate problems of the supersymmetric coupled brane system it is desirable to have a version of the superstring and superbrane actions which admits a straightforward and explicit lifting to the whole 10–dimensional space or to the whole D9-brane world volume. Fortunately, such a formulation does exist. It is the so-called Lorentz harmonic formulation of the superstring [31] which includes auxiliary moving frame (Lorentz harmonic) variables, treated as worldsheet fields. This is a geometric (in a sense the first-order) action which can be written in terms of differential forms without use of the Hodge operation [14, 15]. The only world volume field which is not an image of a target space one is just the moving frame field (harmonics). However, it is possible to extend this field to an auxiliary 10-dimensional $SO(1,9)/(SO(1,1) \times SO(8)$ 'sigma model', which is subject to the only condition that it should coincide with the 'stringy' harmonics when restricted to the string worldsheet [2].

In this way we obtain a supersymmetric action for the interacting system including super-D9-brane and a fundamental superstring 'ending' on the D9-brane, derive the supersymmetric equations of motion directly from the variation of the action and study different phases of the coupled dynamical system. We shortly discuss as well the generalization of our approach for the case of an arbitrary system of intersecting branes.

For simplicity we are working in flat target $D = 10$ *type IIB* superspace. The generalization to brane systems in arbitrary supergravity *background* is straightforward. Moreover, our

[2] Just the existence of the Lorentz harmonic actions for super–D–branes [32, 33, 34] and super–M–branes [15, 35] guarantees the correctness of the formal approach to the action functional description of interacting bosonic systems [25].

approach allows to involve supergravity in an interacting brane system. To this end one can include a counterpart of the group manifold action for supergravity in the functional describing interacting branes instead of (or together with) the space–time filling brane action.

In Section 2 we consider the peculiar features of an interacting system which contains a space-time filling brane. We describe an induced embedding of the superstring worldsheet into the D9-brane worldvolume. The geometric action [34] and the geometric ('first order') form of the supersymmetric equations of motion for the super–D9–brane are presented in Section 3. Section 4 is devoted to the description of the geometric (twistor–like Lorentz harmonic) action and of the equations of motion for the free type IIB superstring. Here Lorentz harmonic variables are used and the issue of supersymmetry breaking by boundaries is addressed briefly. In Section 5 we introduce the density with support localized on the superstring worldsheet and motivate that it becomes invariant under $D=10$ type IIB supersymmetry when the identification (1.2) is imposed.

The action functional describing the interacting system of the super–D9–brane and the (in general open) fundamental superstring ('ending' on the super-D9-brane) is presented in Section 6. The equations of motion of the interacting system are derived in Section 7 and analyzed in Section 8. The issues of kappa-symmetry and supersymmetry in the coupled system are addressed there. In the last Section we summarize our results and also discuss a generalization of our approach to an arbitrary system of intersecting branes.

2 The space-time filling brane

The embedding of the super–D9–brane worldvolume

$$\mathcal{M}^{1+9} = \{x^m\}, \qquad m = 0, \ldots, 9 \tag{2.1}$$

into the $D=10$ type II target superspace

$$\underline{\mathcal{M}}^{(1+9|32)} = \{X^{\underline{m}}, \Theta^{\underline{\mu}1}, \Theta^{\underline{\mu}2}\}, \qquad \underline{m} = 0, \ldots, 9 \qquad \underline{\mu} = 1, \ldots, 16 \tag{2.2}$$

can be described locally by the coordinate superfunctions

$$X^{\underline{m}} = X^{\underline{m}}(x^m), \qquad \Theta^{I\underline{\mu}} = \Theta^{I\underline{\mu}}(x^m), \quad I = 1, 2. \tag{2.3}$$

In addition, there is an intrinsic world volume gauge field living on the D9-brane world volume

$$A = dx^m A_m(x^n). \tag{2.4}$$

For nonsingular D9-brane configurations the function $X^{\underline{m}}(x^m)$ should be assumed to be nondegenerate in the sense $det\,(\partial_n X^{\underline{m}}(x^m)) \neq 0$. Thus the inverse function

$$x^m = x^m(X^{\underline{m}}) \tag{2.5}$$

does exist and, hence, the Grassmann coordinate functions (2.3) and the Born-Infeld gauge field (2.4) can be considered as functions of $X^{\underline{m}}$ variables. In this manner an alternative parametrization of the D9-brane world volume is provided by

$$\mathcal{M}^{1+9} \;\rightarrow\; \underline{\mathcal{M}}^{(1+9|32)}: \qquad \mathcal{M}^{1+9} = \{(X^{\underline{m}}, \Theta^{I\underline{\mu}}(X^{\underline{m}}))\}, \qquad A = dX^{\underline{m}} A_{\underline{m}}(X^{\underline{n}}), \tag{2.6}$$

which clarifies the fact that the $D = 10$, type II super–D9–brane is a theory of Volkov-Akulov Goldstone fermion [36] combined into a supermultiplet with the vector field $A_{\underline{m}}(X^{\underline{n}})$ (see [34]).

Through the intermediate step (2.5), (2.6) we can define the *induced embedding of the superstring worldsheet into the D9-brane world volume*.

2.1 Induced embedding of the superstring worldsheet

Indeed, the embedding of the fundamental superstring worldsheet

$$\mathcal{M}^{1+1} = \{\xi^{(\pm\pm)}\} = \{\xi^{(++)}, \xi^{(--)}\}, \qquad \xi^{(++)} = \tau + \sigma, \qquad \xi^{(--)} = \tau - \sigma, \tag{2.7}$$

into the $D = 10$ type IIB target superspace $\underline{\mathcal{M}}^{(1+9|32)}$ (2.2) can be described locally by the coordinate superfunctions

$$X^{\underline{m}} = \hat{X}^{\underline{m}}(\xi^{(\pm\pm)}), \qquad \Theta^{I\underline{\mu}} = \hat{\Theta}^{I\underline{\mu}}(\xi^{(\pm\pm)}), \quad I = 1, 2. \tag{2.8}$$

However, using the existence of the inverse function (2.5), one can define the *induced* embedding of the worldsheet into the D9-brane world volume

$$x^m = x^m(\xi) \equiv x^m\left(\hat{X}^{\underline{m}}(\xi)\right). \tag{2.9}$$

As superstring and super–D9–brane live in the same $D = 10$ type IIB superspace, we can use the identification of the Grassmann coordinate fields of the superstring with the images of the Grassmann coordinate fields of the super–D9–brane (Goldstone fermions) on the worldsheet

$$\hat{\Theta}^{I\underline{\mu}}(\xi^{(\pm\pm)}) = \Theta^{I\underline{\mu}}(\hat{X}^{\underline{m}}(\xi^{(\pm\pm)})), \tag{2.10}$$

or, equivalently,

$$\hat{X}^{\underline{m}}(\xi^{(\pm\pm)}) = \hat{X}^{\underline{m}}\left(x^m(\xi^{(\pm\pm)})\right), \qquad \hat{\Theta}^{I\underline{\mu}}(\xi^{(\pm\pm)}) = \Theta^{I\underline{\mu}}\left(x^m(\xi^{(\pm\pm)})\right), \tag{2.11}$$

to study the interaction of the fundamental superstring with the super-D9-brane.

The approach based on such an identification was called 'Goldstone fermion embedded' (GFE) in [22] because, from another viewpoint, the superstring worldsheet can be regarded as embedded into Goldstone fermion theory rather than into superspace.

2.2 Tangent and cotangent space.

The pull-backs of the basic forms (flat supervielbeine) of flat $D = 10$ type IIB superspace

$$E^{\underline{a}} = \Pi^{\underline{m}} u_{\underline{m}}^{\ \underline{a}} \equiv (dX^{\underline{m}} - id\Theta^1\sigma^{\underline{m}}\Theta^1 - id\Theta^2\sigma^{\underline{m}}\Theta^2) u_{\underline{m}}^{\ \underline{a}}, \tag{2.12}$$

$$E^{\underline{\alpha}1} = d\Theta^{1\underline{\mu}} v_{\underline{\mu}}^{\ \underline{\alpha}}, \qquad E^{\underline{\alpha}2} = d\Theta^{2\underline{\mu}} v_{\underline{\mu}}^{\ \underline{\alpha}} \tag{2.13}$$

to the D9-brane worldvolume are defined by the decomposition on the holonomic basis dx^m or $dX^{\underline{m}}$

$$dX^{\underline{m}} = dx^m \partial_m X^{\underline{m}}(x). \tag{2.14}$$

The basic relations are

$$\Pi^{\underline{m}} = dX^{\underline{n}}\Pi_{\underline{n}}^{\ \underline{m}} = dx^m \Pi_m^{\ \underline{m}}, \tag{2.15}$$

$$\Pi_{\underline{n}}^{\ \underline{m}} \equiv \delta_{\underline{n}}^{\underline{m}} - i\partial_{\underline{n}}\Theta^1\sigma^{\underline{m}}\Theta^1 - i\partial_{\underline{n}}\Theta^2\sigma^{\underline{m}}\Theta^2, \tag{2.16}$$

$$\Pi_n^{\ \underline{m}} \equiv \partial_n X^{\underline{m}} - i\partial_n\Theta^1\sigma^{\underline{m}}\Theta^1 - i\partial_n\Theta^2\sigma^{\underline{m}}\Theta^2, \tag{2.17}$$

$$d\Theta^{\underline{\mu}I} = dX^{\ \underline{m}}\partial_{\underline{m}}\Theta^{\underline{\mu}I} = dx^m\partial_m\Theta^{\underline{\mu}I}(x^n), \tag{2.18}$$

$$\partial_m\Theta^{\underline{\mu}I}(x^n) \equiv \partial_m X^{\ \underline{m}}(x)\partial_{\underline{m}}\Theta^{\underline{\mu}I}. \tag{2.19}$$

The matrices $u_{\underline{m}}^{\underline{a}}$, $v_{\underline{\mu}}^{\underline{\alpha}}$, involved into Eqs. (2.12), (2.13) take their values in the Lorentz group

$$u_{\underline{m}}^{\underline{a}} \qquad \in SO(1, D-1), \tag{2.20}$$

and in its doubly covering group $Spin(1, D-1)$

$$v_{\underline{\mu}}^{\underline{\alpha}} \qquad \in Spin(1, D-1), \tag{2.21}$$

respectively and represent the same Lorentz transformations. The latter imply the relations

$$u_{\underline{m}}^{\underline{a}}\tilde{\sigma}_{\underline{a}}^{\underline{\alpha\beta}} = v_{\underline{\mu}}^{\underline{\alpha}}\tilde{\sigma}_{\underline{a}}^{\underline{\mu\nu}}v_{\underline{\nu}}^{\underline{\beta}}, \qquad u_{\underline{m}}^{\underline{a}}\tilde{\sigma}_{\underline{\mu\nu}}^{\underline{m}} = v_{\underline{\mu}}^{\underline{\alpha}}\tilde{\sigma}_{\underline{\alpha\beta}}^{\underline{m}}v_{\underline{\nu}}^{\underline{\beta}}, \tag{2.22}$$

between these matrices which reflect the invariance of the $D = 10$ sigma matrices under the Lorentz group transformations (see [31, 14]). The variables (2.20), (2.21) are not necessary for the description of the super–D9–brane itself. However, as we shall see below, they are useful for the description of the coupled system including a brane 'ending' on (interacting with) the D9-brane.

For that system it is important to note that the pull-backs of bosonic supervielbein forms $\Pi^{\underline{m}}$ or $E^{\underline{a}}$ of type IIB superspace can be used as a basis in the space cotangent to the world-volume of D9–brane. In other words, it is convenient to use the invertibility of the matrix $\Pi_n^{\ \underline{m}}$ (2.17) and the harmonic variables to define the covariant basis $\nabla_{\underline{m}}$ and the one $\nabla_{\underline{a}}$ of the space tangent to the D9-brane worldvolume by

$$d \equiv dx^n\partial_m = dX^{\underline{m}}\partial_{\underline{m}} = \Pi^{\underline{m}}\nabla_{\underline{m}} = E^{\underline{a}}\nabla_{\underline{a}}, \tag{2.23}$$

$$\nabla_{\underline{m}} = \Pi^{-1}{}_{\underline{m}}^{\ \underline{n}}\partial_{\underline{n}} = \Pi^{-1n}{}_{\underline{m}}\partial_n, \qquad \nabla_{\underline{a}} \equiv u_{\underline{a}}^{\ \underline{m}}\nabla_{\underline{m}}. \tag{2.24}$$

3 Geometric action and equations of motion for super-D9-brane

3.1 Geometric action

The geometric action for the super–D9–brane in flat $D = 10$, type IIB superspace is [34]

$$S = \int_{\mathcal{M}^{10}} \mathcal{L}_{10} = \int_{\mathcal{M}^{10}} (\mathcal{L}_{10}^0 + \mathcal{L}_{10}^1 + \mathcal{L}_{10}^{WZ}) \tag{3.1}$$

where

$$\mathcal{L}_{10}^0 = \Pi^{\wedge 10}|\eta + F| \tag{3.2}$$

with

$$|\eta + F| \equiv \sqrt{-det(\eta_{\underline{mn}} + F_{\underline{mn}})},$$

$$\Pi^{\wedge 10} \equiv \frac{1}{(10)!}\epsilon_{\underline{m}_1 \ldots \underline{m}_{10}} \Pi^{\underline{m}_1} \wedge \ldots \wedge \Pi^{\underline{m}_{10}}, \tag{3.3}$$

$$\mathcal{L}^1_{10} = Q_8 \wedge (dA - B_2 - \frac{1}{2}\Pi^{\underline{m}} \wedge \Pi^{\underline{n}} F_{\underline{nm}}). \tag{3.4}$$

$A = dx^m A_m(x)$ is the gauge field inherent to the Dirichlet branes, B_2 represents the NS–NS gauge field with flat superspace value

$$B_2 = i\Pi^{\underline{m}} \wedge \left(d\Theta^1 \sigma_{\underline{m}} \Theta^1 - d\Theta^2 \sigma_{\underline{m}} \Theta^2 \right) + d\Theta^1 \sigma^{\underline{m}} \Theta^1 \wedge d\Theta^2 \sigma_{\underline{m}} \Theta^2 \tag{3.5}$$

and field strength

$$H_3 = dB_2 = i\Pi^{\underline{m}} \wedge \left(d\Theta^1 \sigma_{\underline{m}} \wedge d\Theta^1 - d\Theta^2 \sigma_{\underline{m}} \wedge d\Theta^2 \right). \tag{3.6}$$

The Wess-Zumino Lagrangian form is the same as the one appearing in the standard formulation [37, 38, 39, 40, 41]

$$\mathcal{L}^{WZ}_{10} = e^{\mathcal{F}_2} \wedge C|_{10}, \qquad C = \oplus^5_{n=0} C_{2n}, \qquad e^{\mathcal{F}_2} = \oplus^5_{n=0} \frac{1}{n!} \mathcal{F}_2^{\wedge n}, \tag{3.7}$$

where the formal sum of the RR superforms $C = C_0 + C_2 + \ldots$ and of the external powers of the 2–form

$$\mathcal{F}_2 \equiv dA - B_2 \tag{3.8}$$

(e.g. $\mathcal{F}^{\wedge 2} \equiv \mathcal{F} \wedge \mathcal{F}$ etc.) is used and $|_{10}$ means the restriction to the 10–superform input. Let us note that the restriction of the same expression (3.7) to the $(p+1)$–form input (where $p = 2k-1$ is odd)

$$\mathcal{L}^{WZ}_{p+1} = e^{\mathcal{F}} \wedge C|_{p+1} = \oplus^5_{n=0} C_{2n} \wedge \oplus^5_{n=0} \frac{1}{n!} \mathcal{F}^{\wedge n}|_{p+1} \tag{3.9}$$

describes the Wess-Zumino term of the super-Dp-brane of type IIB theory [37, 39, 41]. This will be important for the description of the supersymmetric generalization of the Born-Infeld equations for the D9-brane gauge fields, where the D7-brane Wess-Zumino term appears.

For most applications only the external derivative of the Wess-Zumino term is important. It has the form

$$d\mathcal{L}^{WZ}_{10} = e^{\mathcal{F}} \wedge R|_{11}, \qquad R = \oplus^5_{n=0} R_{2n+1}, \tag{3.10}$$

with the 'vacuum' (i.e. flat target superspace) values of the Ramond-Ramond curvatures specified as

$$R = \oplus^5_{n=0} R_{2n+1} = e^{-\mathcal{F}} \wedge d(e^{\mathcal{F}} \wedge C) = 2id\Theta^{2\underline{\nu}} \wedge d\Theta^{1\underline{\mu}} \wedge \oplus^4_{n=0} \hat{\sigma}^{(2n+1)}_{\underline{\nu\mu}}. \tag{3.11}$$

In the action variations and expressions for currents the notion of 'dual' forms

$$\begin{aligned}
&\Pi^{\wedge 9}_{\underline{m}} \equiv \frac{1}{9!}\epsilon_{\underline{m}\underline{m}_1 \ldots \underline{m}_9} \Pi^{\underline{m}_1} \wedge \ldots \wedge \Pi^{\underline{m}_9}, \\
&\Pi^{\wedge 8}_{\underline{mn}} \equiv \frac{1}{2 \cdot 8!}\epsilon_{\underline{mn}\underline{m}_1 \ldots \underline{m}_8} \Pi^{\underline{m}_1} \wedge \ldots \wedge \Pi^{\underline{m}_8}, \qquad \ldots \\
&\Pi^{\wedge (10-k)}_{\underline{m}_1 \ldots \underline{m}_k} \equiv \frac{1}{k!(10-k)!}\epsilon_{\underline{m}_1 \ldots \underline{m}_k \underline{n}_1 \ldots \underline{n}_{(10-k)}} \Pi^{\underline{n}_1} \wedge \ldots \wedge \Pi^{\underline{n}_{(10-k)}}
\end{aligned} \tag{3.12}$$

is useful. The list of products of the forms (3.12) includes the useful identities

$$\begin{gathered}\Pi^{\wedge 9}_{\underline{m}} \wedge \Pi^{\underline{n}} = -\Pi^{\wedge 10}\delta^{\underline{n}}_{\underline{m}}, \qquad \Pi^{\wedge 8}_{\underline{mn}} \wedge \Pi^{\underline{l}} = \Pi^{\wedge 9}_{[\underline{m}}\delta^{\underline{l}}_{\underline{n}]}, \\ \Pi^{\wedge 7}_{\underline{mnk}} \wedge \Pi^{\underline{l}} = -\Pi^{\wedge 8}_{[\underline{mn}}\delta^{\underline{l}}_{\underline{k}]}, \Pi^{\wedge 6}_{\underline{mnkl}} \wedge \Pi^{\underline{r}} = \Pi^{\wedge 7}_{[\underline{mnk}}\delta^{\underline{r}}_{\underline{l}]}, \\ \Pi^{\wedge 6}_{\underline{mnkl}} \wedge \Pi^{\underline{r}} \wedge \Pi^{\underline{s}} = \Pi^{\wedge 8}_{[\underline{mn}}\delta^{\underline{r}}_{\underline{k}}\delta^{\underline{s}}_{\underline{l}]}.\end{gathered} \tag{3.13}$$

3.2 Variation of geometrical action for D9-brane

The simplest way to vary the geometrical action (3.1)–(3.4) starts by taking the external derivative of the Lagrangian form $\mathcal{L}_{10}$ (cf. [32, 34])

$$d\mathcal{L}_{10} = \left(dQ_8 + d\mathcal{L}^{WZ}_8|_{\mathcal{F}_2\to F_2}\right) \wedge (\mathcal{F}_2 - F_2) + \tag{3.14}$$

$$\begin{gathered}+(Q_8 - \Pi^{\wedge 8}_{\underline{nm}}\sqrt{|\eta+F|}(\eta+F)^{-1\,\underline{nm}}) \wedge \Big(-\frac{1}{2}\Pi^{\underline{m}} \wedge \Pi^{\underline{n}} \wedge dF_{\underline{nm}} - i\Pi^{\underline{m}} \wedge (d\Theta^1\sigma^{\underline{n}} \wedge d\Theta^1)(\eta - F)_{\underline{nm}} + \\ +i\Pi^{\underline{m}} \wedge (d\Theta^2\sigma^{\underline{n}} \wedge d\Theta^2)(\eta + F)_{\underline{nm}}\Big) \\ +i\Pi^{\wedge 9}_{\underline{m}}\sqrt{|\eta+F|}(\eta+F)^{-1\,\underline{mn}}\sigma_{\underline{n}\underline{\mu}\underline{\nu}} \wedge \left(d\Theta^{2\underline{\mu}} - d\Theta^{1\underline{\rho}}h_{\underline{\rho}}{}^{\underline{\mu}}\right) \wedge \left(d\Theta^{2\underline{\nu}} - d\Theta^{1\underline{\sigma}}h_{\underline{\sigma}}{}^{\underline{\nu}}\right) + \\ +\mathcal{O}\left((\mathcal{F}_2 - F_2)^{\wedge 2})\right),\end{gathered}$$

where $\mathcal{F}_2 \equiv dA - B_2$ (3.8) and $F_2 \equiv \frac{1}{2}\Pi^{\underline{m}} \wedge \Pi^{\underline{n}}F_{\underline{nm}}$. Note that $\mathcal{F}_2 - F_2$ vanishes due to the algebraic equation which is implied by the Lagrange multiplier Q_8. This is the reason why the terms proportional to the second and higher (external) powers of $(\mathcal{F}_2 - F_2)$ are indicated by $\mathcal{O}\left((\mathcal{F}_2 - F_2)^{\wedge 2})\right)$ but not written explicitly.

Then we can use the seminal formula

$$\delta\mathcal{L}_{10} = i_\delta d\mathcal{L}_{10} + d(i_\delta\mathcal{L}_{10}) \tag{3.15}$$

(usually applied for coordinate variations only) supplemented by the formal definition of the contraction with variation symbol

$$i_\delta d\Theta^{1,2\underline{\nu}} = \delta\Theta^{1,2\underline{\nu}}, \quad i_\delta\Pi^{\underline{m}} = \delta X^{\underline{m}} - i\delta\Theta^1\Gamma^{\underline{m}}\Theta^1 - i\delta\Theta^2\Gamma^{\underline{m}}\Theta^2, \tag{3.16}$$

$$i_\delta dA = \delta A, \qquad i_\delta dQ_8 = \delta Q_8, \qquad i_\delta dF_{\underline{mn}} = \delta F_{\underline{mn}}, \qquad \ldots . \tag{3.17}$$

To simplify the algebraic calculations, one notes that it is sufficient to write such a formal contraction modulo terms proportional to the square of the algebraic equations (the latter remains the same for the coupled system as well, because the auxiliary fields, e.g. Q_8, do not appear in the action of other branes):

$$\begin{aligned}\delta S_{D9} = &\int_{\mathcal{M}^{1+9}} (Q_8 - \Pi^{\wedge 8}_{\underline{kl}}\sqrt{|\eta+F|}(\eta+F)^{-1\,\underline{kl}}) \wedge \Big(-\frac{1}{2}\Pi^{\underline{m}} \wedge \Pi^{\underline{n}} \wedge \delta F_{\underline{nm}} + \ldots\Big) + \\ &+ \int_{\mathcal{M}^{1+9}} (\delta Q_8 + \ldots) \wedge (\mathcal{F}_2 - F_2) + \\ &+ \int_{\mathcal{M}^{1+9}} \left(dQ_8 + d\mathcal{L}^{WZ}_8|_{\mathcal{F}_2\to F_2}\right) \wedge (\delta A - i_\delta B_2 + \Pi^{\underline{n}}F_{\underline{nm}}i_\delta\Pi^{\underline{m}}) + \\ &+2i\int_{\mathcal{M}^{1+9}} \Pi^{\wedge 9}_{\underline{m}}\sqrt{|\eta+F|}(\eta+F)^{-1\,\underline{mn}}\sigma_{\underline{n}\underline{\mu}\underline{\nu}} \wedge \left(d\Theta^{2\underline{\mu}} - d\Theta^{1\underline{\rho}}h_{\underline{\rho}}{}^{\underline{\mu}}\right) \ \left(\delta\Theta^{2\underline{\nu}} - \delta\Theta^{1\underline{\sigma}}h_{\underline{\sigma}}{}^{\underline{\nu}}\right) + \\ &+i\int_{\mathcal{M}^{1+9}} \Pi^{\wedge 8}_{\underline{km}}\sqrt{|\eta+F|}(\eta+F)^{-1\,\underline{kn}}\sigma_{\underline{n}\underline{\mu}\underline{\nu}} \wedge \left(d\Theta^{2\underline{\mu}} - d\Theta^{1\underline{\rho}}h_{\underline{\rho}}{}^{\underline{\mu}}\right) \wedge \left(d\Theta^{2\underline{\nu}} - d\Theta^{1\underline{\sigma}}h_{\underline{\sigma}}{}^{\underline{\nu}}\right) i_\delta\Pi^{\underline{m}}\end{aligned} \tag{3.18}$$

Here the terms denoted by ... produce contributions to the equations of motion which are proportional to the algebraic equations and are thus inessential.

The spin-tensor matrix $h_{\underline{\mu}}{}^{\underline{\nu}}$ entering Eqs. (3.18) is related to the antisymmetric tensor $F_{\underline{nm}}$ by the Cayley image relations

$$h_{\underline{\mu}}{}^{\underline{\nu}} \qquad \in \qquad Spin(1,9), \tag{3.19}$$

$$(h\sigma^{\underline{m}}h^T)_{\underline{\mu\nu}} = \sigma^{\underline{n}}_{\underline{\mu\nu}} k_{\underline{n}}{}^{\underline{m}} \equiv \sigma^{\underline{n}}_{\underline{\mu\nu}} (\eta+F)^{-1}_{\underline{nl}} (\eta - F)^{\underline{lm}}, \tag{3.20}$$

$$k_{\underline{n}}{}^{\underline{m}} = (\eta+F)^{-1}_{\underline{nl}} (\eta - F)^{\underline{lm}} \equiv (\eta - F)_{\underline{nl}} (\eta+F)^{-1\ \underline{lm}} \qquad \in \qquad SO(1,9). \tag{3.21}$$

For more details we refer to [34].

It is important that δA enters the compact expression (3.18) for the variation of the super–D9–brane action only in the combination

$$i_\delta \left(\mathcal{F}_2 - F_2\right) \equiv \left(\delta A - i_\delta B_2 + \Pi^{\underline{n}} F_{\underline{nm}} i_\delta \Pi^{\underline{m}}\right). \tag{3.22}$$

It can be called a supersymmetric variation of the gauge field as the condition $i_\delta \left(\mathcal{F}_2 - F_2\right) = 0$ actually determines the supersymmetric transformations of the gauge fields (cf. [37, 32, 34]). Together with (3.16), the expression (3.22) defines the basis of supersymmetric variations, whose use simplifies in an essential manner the form of the equations of motion.

The formal external derivative of the Lagrangian form (3.14) can be used as well for the general coordinate variation of the action (3.1)-(3.4)

$$\delta S_{D9} = \int_{\mathcal{M}^{1+9}} \delta x^m i_m d\mathcal{L}_{10}, \tag{3.23}$$

where for any q-form Q_q the operation i_m is defined by

$$Q_q = \frac{1}{q!} dx^{m_1} \wedge \ldots \wedge dx^{m_q} Q_{m_q \ldots m_1} \qquad i_m Q_q = \frac{1}{(q-1)!} dx^{m_1} \wedge \ldots \wedge dx^{m_{q-1}} Q_{m m_{q-1} \ldots m_1}. \tag{3.24}$$

For the free super–D9–brane such a variation vanishes identically when the 'field' equations of motion are taking into account. This reflects the evident diffeomorphism invariance of the action (3.1). It is not essential as well in the study of coupled branes in the present approach, while in another approach for the description of coupled superbranes [22] such variations play an important role.

3.3 Equations of motion for super–D9–brane

The equations of motion from the geometric action (3.1)–(3.4) split into the algebraic ones obtained from the variation of auxiliary fields Q_8 and $F_{\underline{mn}}$

$$\mathcal{F}_2 \equiv dA - B_2 = F_2 \equiv \frac{1}{2} \Pi^{\underline{m}} \wedge \Pi^{\underline{n}} F_{\underline{nm}}, \tag{3.25}$$

$$Q_8 = \Pi^{\wedge 8}_{\underline{nm}} \sqrt{|\eta + F|} (\eta+F)^{-1\ \underline{nm}} \tag{3.26}$$

and the dynamical ones

$$dQ_8 + d\mathcal{L}_8^{WZ-D7} = 0, \tag{3.27}$$

$$\Pi^{\wedge 9}_{\underline{m}} \sqrt{|\eta + F|} (\eta + F)^{-1\ \underline{mn}} \sigma_{\underline{n}\underline{\mu}\underline{\nu}} \wedge \left(d\Theta^{2\underline{\nu}} - d\Theta^{1\underline{\rho}} h_{\underline{\rho}}^{\ \underline{\nu}} \right) = 0. \tag{3.28}$$

If one takes into account the expression for Q_8 (3.26), the identification of F with the gauge field strength (3.25), as well as the expression for the D7-brane Wess-Zumino term

$$d\mathcal{L}_8^{WZ-D7} = e^{\mathcal{F}} \wedge R|_9, \qquad R = \oplus_{n=0}^{5} R_{2n+1} = 2i d\Theta^{2\underline{\nu}} \wedge d\Theta^{1\underline{\mu}} \wedge \oplus_{n=0}^{4} \hat{\sigma}^{(2n+1)}_{\underline{\nu}\underline{\mu}}, \tag{3.29}$$

one finds that (3.27) is just the supersymmetrized Born-Infeld equation.

The fermionic equations (3.28) appear as a result of the variation with respect to Θ^2, while the variation with respect to Θ^1 does not produce any independent equations. This fact reflects the Noether identity corresponding to the local fermionic κ–symmetry of the super–D9–brane action (3.1) [34].

The explicit *irreducible* form of the D9-brane κ-symmetry transformation can be written with the help of the spin–tensor field h (3.19) – (3.21) [34] as

$$\delta\Theta^{1\underline{\mu}} = \kappa^{\underline{\mu}}, \qquad \delta\Theta^{2\underline{\mu}} = \kappa^{\underline{\nu}} h_{\underline{\nu}}^{\ \underline{\mu}} \tag{3.30}$$

$$i_\delta \Pi^{\underline{m}} = 0, \qquad \Leftrightarrow \qquad \delta X^{\underline{m}} = i\delta\Theta^1 \sigma^{\underline{m}} \Theta^1 - i\delta\Theta^2 \sigma^{\underline{m}} \Theta^2,$$

$$\begin{aligned} & i_\delta \mathcal{F} = 0 \quad \Leftrightarrow \\ & \delta A = i_\delta B_2 \equiv i\Pi^{\underline{m}} \wedge \left(\delta\Theta^1 \sigma_{\underline{m}} \wedge \Theta^1 - \delta\Theta^2 \sigma_{\underline{m}} \wedge \Theta^2 \right) + \\ & + d\Theta^1 \sigma^{\underline{m}} \Theta^1 \wedge \delta\Theta^2 \sigma_{\underline{m}} \Theta^2 - \delta\Theta^1 \sigma^{\underline{m}} \Theta^1 \wedge d\Theta^2 \sigma_{\underline{m}} \Theta^2, \end{aligned} \tag{3.31}$$

$$\delta F_{\underline{mn}} = 2i(\eta - F)_{\underline{l}[\underline{m}} \left(\nabla_{\underline{n}]} \Theta^1\ \sigma^{\underline{l}} \delta\Theta^1 - \nabla_{\underline{n}]} \Theta^2\ \sigma^{\underline{l}} \delta\Theta^2 \right), \qquad \delta Q_8 = 0.$$

The Noether identity reflecting the evident diffeomorphism invariance of the action (3.1) is the dependence of the equations obtained by varying the action (3.1) with respect to $X^{\underline{m}}(x)$

$$i\Pi^{\wedge 8}_{\underline{nm}} \sqrt{|\eta + F|} (\eta + F)^{-1\ \underline{nk}} \sigma_{\underline{k}\underline{\mu}\underline{\nu}} \wedge \left(d\Theta^{2\underline{\nu}} - d\Theta^{1\underline{\rho}} h_{\underline{\rho}}^{\ \underline{\nu}} \right) \wedge \left(d\Theta^{2\underline{\nu}} - d\Theta^{1\underline{\sigma}} h_{\underline{\sigma}}^{\ \underline{\nu}} \right) = 0. \tag{3.32}$$

Indeed it can be proved that Eq. (3.32) is satisfied identically, when Eq.(3.28) is taken into account.

Turning back to the fermionic equations (3.28), let us note that after decomposition

$$d\Theta^{2\underline{\nu}} - d\Theta^{1\underline{\rho}} h_{\underline{\rho}}^{\ \underline{\nu}} = \Pi^{\underline{m}} \Psi_{\underline{m}}^{\ \underline{\nu}}, \tag{3.33}$$

where

$$\Psi_{\underline{m}}^{\ \underline{\nu}} = \nabla_{\underline{m}} \Theta^{2\underline{\nu}} - \nabla_{\underline{m}} \Theta^{1\underline{\rho}} h_{\underline{\rho}}^{\ \underline{\nu}} \tag{3.34}$$

and $\nabla_{\underline{m}}$ defined by $d = dx^m \partial_m = \Pi^{\underline{m}} \nabla_{\underline{m}}$ (2.23), (2.24), one arrives for (3.28) at [34]

$$-i\Pi^{\wedge 10} \sqrt{|\eta + F|} \sigma_{\underline{k}\underline{\mu}\underline{\nu}} \Psi_{\underline{m}}^{\ \underline{\nu}} (\eta + F)^{-1\ \underline{mn}} = 0, \qquad \Leftrightarrow \qquad \sigma_{\underline{k}\underline{\mu}\underline{\nu}} \Psi_{\underline{m}}^{\ \underline{\nu}} (\eta + F)^{-1\ \underline{mn}} = 0. \tag{3.35}$$

4 Geometric action and free equations of motion for type IIB superstring

4.1 Geometric action and moving frame variables (Lorentz harmonics)

In the geometric action for type IIB superstring [31, 14, 15]

$$S_{IIB} = \int\limits_{\mathcal{M}^{(1+1)}} \hat{\mathcal{L}}_2^{IIB} = \int\limits_{\mathcal{M}^{(1+1)}} \left(\frac{1}{2}\hat{E}^{++} \wedge \hat{E}^{--} - \hat{B}_2\right), \tag{4.1}$$

the hat (as in (4.1)) indicates the fields restricted to (or living on) a superstring worldsheet (2.7) $\mathcal{M}^{(1+1)} = \{\xi^{(\pm\pm)}\}$. $\hat{B}_2$ is the pull-back of the NS-NS gauge field with the 'vacuum' (i.e. flat superspace) value (3.5) which plays the role of the Wess-Zumino term in the superstring action, furthermore

$$\hat{E}^{\pm\pm} = \hat{\Pi}^{\underline{m}} \hat{u}_{\underline{m}}^{\pm\pm}, \tag{4.2}$$

where $\hat{u}_{\underline{m}}^{\pm\pm}(\xi)$ are vector harmonics [45, 31, 14, 15], i.e. two light–like vector fields entering the $SO(1,9)$ valued matrix (2.20)

$$\hat{u}_{\underline{m}}^{\underline{a}}(\xi) \equiv (\hat{u}_{\underline{m}}^{++}, \hat{u}_{\underline{m}}^{--}, \hat{u}_{\underline{m}}^{i}) \qquad \in \qquad SO(1, D-1). \tag{4.3}$$

This matrix describes a moving frame attached to the worldsheet and thus provides the possibility to adapt the general bosonic vielbein of the flat superspace to the embedding of the worldsheet

$$\hat{E}^{\underline{a}} = \left(\hat{E}^{++}, \hat{E}^{--}, \hat{E}^{i}\right) \equiv \hat{\Pi}^{\underline{m}} \hat{u}_{\underline{m}}^{\underline{a}}. \tag{4.4}$$

The properties of the harmonics (4.3) are collected in the Appendix A. To obtain equations of motion from the geometric action (4.1) it is important that the variations of the light-like harmonics $\hat{u}_{\underline{m}}^{\pm\pm}$ should be performed with the constraint (4.3), i.e. with

$$\hat{u}_{\underline{m}}^{\underline{a}} \eta_{\underline{mn}} \hat{u}_{\underline{n}}^{\underline{b}} = \eta^{\underline{ab}} \quad \Rightarrow \quad \begin{cases} u_{\underline{m}}^{++} u^{++\underline{m}} = 0, & u_{\underline{m}}^{--} u^{--\underline{m}} = 0, \\ u_{\underline{m}}^{\;i} u^{++\underline{m}} = 0, & u_{\underline{m}}^{i} u^{--\underline{m}} = 0, \\ u_{\underline{m}}^{++} u^{--\underline{m}} = 2, & u_{\underline{m}}^{i} u^{j\underline{m}} = -\delta^{ij} \end{cases}, \tag{4.5}$$

taken into account. The simplest way to implement this consists in solving the conditions of the conservation of the constraints (4.5)

$$\delta \hat{u}_{\underline{m}}^{\underline{a}} \eta_{\underline{mn}} \hat{u}_{\underline{n}}^{\;\underline{b}} + \hat{u}_{\underline{m}}^{\underline{a}} \eta_{\underline{mn}} \delta \hat{u}_{\underline{n}}^{\;\underline{b}} = 0$$

with respect to $\delta \hat{u}_{\underline{n}}^{\;\underline{b}}$ and, thus, to define a set of variations ('admissible variations' [31]) [3] which then shall be treated as independent. Some of those variations $i_\delta f^{++i}$, $i_\delta f^{--i}$, $i_\delta \omega$ enter the expression for the admissible variations of the light-like harmonics [31]

$$\delta u_{\underline{m}}^{++} = u_{\underline{m}}^{++} i_\delta \omega + \hat{u}_{\underline{m}}^{i} i_\delta f^{++i}, \qquad \delta u_{\underline{m}}^{--} = -\hat{u}_{\underline{m}}^{--} i_\delta \omega + u_{\underline{m}}^{i} i_\delta f^{--i}, \tag{4.6}$$

[3] This is the place to note that a similar technique was used ([42] and refs. therein) in the study of the G/H sigma model fields, appearing in the maximal $D = 3, 4, 5$ supergravities ($G/H = E_{8(+8)}/SO(16)$, $E_{7(+7)}/SU(8)$, $E_{6(+6)}/USp(8)$).

while the other $i_\delta A^{ij}$ are involved in the variations of the orthogonal components of a moving frame

$$\delta u^i_{\underline{m}} = -u^j_{\underline{m}} i_\delta A^{ji} + \frac{1}{2} u^{++}_{\underline{m}} i_\delta f^{--i} + \frac{1}{2} u^{--}_{\underline{m}} i_\delta f^{++i}(d) \tag{4.7}$$

only and thus produce no inputs into the variation of the action (4.1).

The derivatives of the harmonic variables should dealt with in the same way.

4.2 Action variation and equations of motion

The external derivative of the Lagrangian form $\mathcal{L}_2$ is

$$d\mathcal{L}^{IIB}_2 = -2iE^{++} \wedge E^{-1}_{\dot q} \wedge E^{-1}_{\dot q} + 2iE^{--} \wedge E^{+2}_q \wedge E^{+2}_q + \tag{4.8}$$

$$+\frac{1}{2} E^i \wedge \left(E^{--} \wedge f^{++i} - E^{++} \wedge f^{--i} + 4i(E^{+1}_q \wedge E^{-2}_{\dot q} - E^{+2}_q \wedge E^{-1}_{\dot q})\gamma^i_{q\dot q} \right).$$

Here

$$f^{++i} \equiv u^{++}_{\underline{m}} du^{\underline{m}i}, \qquad f^{--i} \equiv u^{--}_{\underline{m}} du^{\underline{m}i}, \tag{4.9}$$

$$\omega \equiv \frac{1}{2} u^{--}_{\underline{m}} du^{\underline{m}++}, \qquad A^{ij} \equiv u^i_{\underline{m}} du^{\underline{m}j}, \tag{4.10}$$

are Cartan forms [31, 14] (see Appendix A) and

$$\hat{E}^{\underline{\alpha}I} \equiv d\hat{\Theta}^{\underline{\mu}I} \hat{v}^{\underline{\alpha}}_{\underline{\mu}} = \left(\hat{E}^{I+}_q, \hat{E}^{I-}_{\dot q} \right) \tag{4.11}$$

$$q = 1, \ldots 8, \qquad \dot q = 1, \ldots 8,$$

are pull-backs of the fermionic supervielbein forms which, together with (4.4), form a basis of the flat target superspace. They involve the spinor harmonics [46, 31]

$$\hat{v}^{\underline{\alpha}}_{\underline{\mu}} = \left(\hat{v}^{I+}_{\underline{\mu}q}, \hat{v}^{I-}_{\underline{\mu}\dot q} \right) \qquad \in \qquad Spin(1,9) \tag{4.12}$$

which represent the same Lorentz rotation (relating the 'coordinate frame' $\Pi^{\underline{m}}, d\Theta^{\underline{\mu}I}$ of the target superspace with the arbitrary frame $E^{\underline{a}}, E^{\underline{\alpha}I}$) as the vector harmonics (4.3) and, hence, are connected with them by Eqs. (2.22). The latter include in particular the relations

$$u^{++}_{\underline{m}} \sigma^{\underline{m}}_{\underline{\mu}\underline{\nu}} = 2v^{~+}_{\underline{\mu}q} v^{~+}_{\underline{\mu}q}, \qquad u^{--}_{\underline{m}} \sigma^{\underline{m}}_{\underline{\mu}\underline{\nu}} = 2v^{~-}_{\underline{\mu}\dot q} v^{~-}_{\underline{\mu}\dot q}, \qquad u^i_{\underline{m}} \sigma^{\underline{m}}_{\underline{\mu}\underline{\nu}} = 2v^{~+}_{\{\underline{\mu}q} \gamma^i_{q\dot q} v^{~-}_{\underline{\nu}\}\dot q}, \tag{4.13}$$

which were used to write $d\mathcal{L}^{IIB}_2$ in a compact form (4.8). For further details concerning harmonics we refer to Appendix A and to the original references [46, 31, 14].

Now one can calculate the variation of the action (4.1) of closed type IIB superstring from the expression (4.8) using the technique described in Section 2.2

$$\delta S_{IIB} = \int_{\mathcal{M}^{(1+1)}} i_\delta d\mathcal{L}^{IIB}_2 = \int_{\mathcal{M}^{(1+1)}} \frac{1}{2} \hat{E}^i \wedge \left(E^{--} i_\delta f^{++i} - E^{++} i_\delta f^{--i} + \ldots \right) +$$

$$\int_{\mathcal{M}^{(1+1)}} \left(\hat{M}^i_2 \; u^i_{\underline{m}} + 2i\hat{E}^{1+}_q \wedge \hat{E}^{1+}_q u^{--}_{\underline{m}} - 2i\hat{E}^{2-}_{\dot q} \wedge \hat{E}^{2-}_{\dot q} u^{++}_{\underline{m}} \right) i_\delta \Pi^{\underline{m}} + \tag{4.14}$$

$$+ \int_{\mathcal{M}^{(1+1)}} \left(-4i\hat{E}^{++} \wedge \hat{E}^{1-}_{\dot q} v^{~-}_{\underline{\mu}\dot q} \delta\Theta^{1\underline{\mu}} + 4i\hat{E}^{--} \wedge \hat{E}^{2+}_q v^{~+}_{\underline{\mu}q} \delta\Theta^{2\underline{\mu}} \right).$$

Here

$$\hat{M}_2^i \equiv \frac{1}{2}\hat{E}^{--} \wedge \hat{f}^{++i} - \frac{1}{2}\hat{E}^{++} \wedge \hat{f}^{--i} + 2i\hat{E}^{1+}_q \wedge \gamma^i_{q\dot{q}}\hat{E}^{1-}_{\dot{q}} - 2i\hat{E}^{2+}_q \wedge \gamma^i_{q\dot{q}}\hat{E}^{2-}_{\dot{q}} \tag{4.15}$$

and the dots in the first line denote the terms

$$f^{++i}i_\delta E^{--} - f^{--i}i_\delta E^{++} + 4i\left(\hat{E}^{1+}_q\gamma^i_{q\dot{q}}\hat{v}^{-}_{\underline{\mu}\dot{q}} + \hat{E}^{1-}_{\dot{q}}\gamma^i_{q\dot{q}}\hat{v}^{+}_{\underline{\mu}q}\right)\delta\Theta^{1\underline{\mu}} - 4i\left(\hat{E}^{2+}_q\gamma^i_{q\dot{q}}\hat{v}^{-}_{\underline{\mu}\dot{q}} + \hat{E}^{2-}_{\dot{q}}\gamma^i_{q\dot{q}}\hat{v}^{+}_{\underline{\mu}q}\right)\delta\Theta^{2\underline{\mu}}$$

which produce contributions proportional to E^i into the action variation. They are essential only when we search for the κ–symmetry of the free type IIB superstring action.

It is worth mentioning that, in contrast to the standard formulation [20], the geometric action (4.1) possesses the *irreducible* κ–symmetry whose transformation is given by (cf. [31, 15])

$$\delta\hat{\Theta}^{\underline{\mu}1} = \kappa^{+q}\hat{v}^{-\underline{\mu}}_{\dot{q}}, \qquad \delta\hat{\Theta}^{\underline{\mu}2} = \kappa^{-\dot{q}}\hat{v}^{+\underline{\mu}}_{q}, \tag{4.16}$$

$$\delta\hat{X}^{\underline{m}} = i\delta\hat{\Theta}^{\underline{\mu}1}\sigma^{\underline{m}}\hat{\Theta}^{\underline{\mu}1} + i\delta\hat{\Theta}^{\underline{\mu}2}\sigma^{\underline{m}}\hat{\Theta}^{\underline{\mu}2}$$

$$\delta\hat{v}^{+}_{\underline{\mu}q} = \frac{1}{2}i_\delta f^{++i}\gamma^i_{q\dot{q}}\hat{v}^{-}_{\underline{\mu}\dot{q}}, \qquad \delta\hat{v}^{-}_{\underline{\mu}\dot{q}} = \frac{1}{2}i_\delta f^{--i}\gamma^i_{q\dot{q}}\hat{v}^{+}_{\underline{\mu}q} \tag{4.17}$$

$$\delta\hat{u}^{++}_{\underline{m}} = \hat{u}^i_{\underline{m}}i_\delta f^{++i}, \qquad \delta\hat{u}^{--}_{\underline{m}} = \hat{u}^i_{\underline{m}}i_\delta f^{--i}, \qquad \delta\hat{u}^i_{\underline{m}} = \frac{1}{2}\hat{u}^{++}_{\underline{m}}i_\delta f^{--i} + \frac{1}{2}\hat{u}^{--}_{\underline{m}}i_\delta f^{++i},$$

(cf. Appendix A) with $i_\delta f^{++i}, i_\delta f^{--i}$ determined by

$$\begin{aligned}&\hat{E}^{++}i_\delta f^{--i} - \hat{E}^{--}i_\delta f^{++i} = \\ &= 4i\left(\hat{E}^{1+}_q\gamma^i_{q\dot{q}}\hat{v}^{-}_{\underline{\mu}\dot{q}} + \hat{E}^{1-}_{\dot{q}}\gamma^i_{q\dot{q}}\hat{v}^{+}_{\underline{\mu}q}\right)\delta\Theta^{1\underline{\mu}} - 4i\left(\hat{E}^{2+}_q\gamma^i_{q\dot{q}}\hat{v}^{-}_{\underline{\mu}\dot{q}} + \hat{E}^{2-}_{\dot{q}}\gamma^i_{q\dot{q}}\hat{v}^{+}_{\underline{\mu}q}\right)\delta\Theta^{2\underline{\mu}}.\end{aligned} \tag{4.18}$$

The equations of motion for the free closed type IIB superstring can be extracted easily from (4.14)

$$\hat{E}^i \equiv \hat{\Pi}^{\underline{m}}\hat{u}^i_{\underline{m}} = 0, \tag{4.19}$$

$$\hat{M}^i_2 = 0, \tag{4.20}$$

$$\hat{E}^{++} \wedge \hat{E}^{1-}_{\dot{q}} = 0, \tag{4.21}$$

$$\hat{E}^{--} \wedge \hat{E}^{2+}_{q} = 0, \tag{4.22}$$

where $\hat{M}^i_2$, $E^{\pm\pm}$, $\hat{E}^{2+}_q$, $E^{1-}_{\dot{q}}$ are defined in (4.15), (4.2), (4.11), respectively.

4.3 Linearized fermionic equations

The proof of equivalence of the Lorentz harmonic formulation (4.1) with the standard action of the Green-Schwarz superstring has been given in [31]. To make this equivalence intuitively evident, let us consider the fermionic equations of motion (4.21), (4.22) in the linearized approximation, fixing a static gauge

$$\hat{X}^{\pm\pm} \equiv X^{\underline{m}}u^{\pm\pm}_{\underline{m}} = \xi^{(\pm\pm)}. \tag{4.23}$$

Moreover, we use the κ–symmetry (4.16) to remove half the components $\hat{\Theta}^{1+}_{q} = \hat{\Theta}^{\underline{\mu}1}\hat{v}_{\underline{\mu}q}^{\ +}$, $\hat{\Theta}^{2-}_{\dot{q}} = \hat{\Theta}^{\underline{\mu}2}\hat{v}_{\underline{\mu}\dot{q}}^{\ -}$, of the Grassmann coordinate fields

$$\hat{\Theta}^{1+}_{q} = \hat{\Theta}^{\underline{\mu}1}\hat{v}_{\underline{\mu}q}^{\ +} = 0, \qquad \hat{\Theta}^{2-}_{\dot{q}} = \hat{\Theta}^{\underline{\mu}2}\hat{v}_{\underline{\mu}\dot{q}}^{\ -} = 0. \tag{4.24}$$

Thus we are left with 8 bosonic and 16 fermionic fields

$$\hat{X}^{i} = \hat{X}^{\underline{\mu}}\hat{u}_{\underline{m}}^{\ i}, \qquad \hat{\Theta}^{1-}_{\dot{q}} = \hat{\Theta}^{\underline{\mu}1}\hat{v}_{\underline{\mu}\dot{q}}^{\ -}, \qquad \hat{\Theta}^{2+}_{q} = \hat{\Theta}^{\underline{\mu}2}\hat{v}_{\underline{\mu}q}^{\ +}. \tag{4.25}$$

In the linearized approximation all the inputs from the derivatives of harmonic variables (i.e. Cartan forms (4.9), (4.10)) disappear from the fermionic equations for the physical Grassmann coordinate fields. Thus we arrive at the counterpart of the gauge fixed string theory in the light–cone gauge. Then it is not hard to see that Eqs. (4.21), (4.22) reduce to the opposite chirality conditions for physical fermionic fields

$$\partial_{--}\hat{\Theta}^{1-}_{\dot{q}} = 0, \qquad \partial_{++}\hat{\Theta}^{2+}_{q} = 0. \tag{4.26}$$

To obtain the bosonic equations, the derivatives of the harmonics (Cartan forms (4.9)) must be taken into account. After exclusion of the auxiliary variables one obtains that Eqs. (4.19), (4.20) reduce to the usual free field equations for 8 bosonic fields X^{i} (see Appendix B for details)

$$\partial_{--}\partial_{++}\hat{X}^{i} = 0. \tag{4.27}$$

4.4 Geometric action with boundary term

To formulate the interaction of the open superstring with the super–D9–brane we have to add to the action (4.1) the boundary term which describes the coupling to the gauge field $A = dx^{m}A_{m}(x)$ inherent to the D9-brane (see (3.4), (3.7), (3.8), (3.9)). Thus the complete action for the open fundamental superstring becomes (cf. [8])

$$S_{I} = S_{IIB} + S_{b} = \int\limits_{\mathcal{M}^{(1+1)}} \hat{\mathcal{L}}_{2} = \int\limits_{\mathcal{M}_{(1+1)}} \left(\frac{1}{2}\hat{E}^{++}\wedge\hat{E}^{--} - \hat{B}_{2}\right) + \int\limits_{\partial\mathcal{M}^{(1+1)}} \hat{A}\ . \tag{4.28}$$

The variation of the action (4.28) differs from $\int i_{\delta}d\mathcal{L}_{2}^{IIB}$ in (4.14) by boundary contributions. In the supersymmetric basis (3.16), (3.22) the variation becomes

$$\begin{aligned}\delta S_{I} = &\int_{\mathcal{M}^{1+1}} i_{\delta}d\mathcal{L}_{2}^{IIB} + \int_{\partial\mathcal{M}^{1+1}} i_{\delta}\left(\mathcal{F}_{2} - F_{2}\right) + \\ &+ \int_{\partial\mathcal{M}^{1+1}} \left(\frac{1}{2}E^{++}u_{\underline{m}}^{--} - \frac{1}{2}E^{--}u_{\underline{m}}^{++} - \Pi^{\underline{m}}F_{\underline{nm}}\right) i_{\delta}\Pi^{\underline{m}}.\end{aligned} \tag{4.29}$$

It is worth mentioning that no boundary contribution with variation $\delta\Theta^{I}$ appears. This does not contradict the well–known fact that the presence of a worldsheet boundary breaks at least a half of the target space $N = 2$ supersymmetry. Indeed, for the supersymmetry transformations

$$\delta_{susy}X^{\underline{m}} = \Theta^{I}\sigma^{\underline{m}}\epsilon^{I}, \qquad \delta_{susy}\Theta^{I\underline{\mu}} = \epsilon^{I\underline{\mu}} \tag{4.30}$$

the variation $i_\delta \Pi^{\underline{m}}$ is nonvanishing and reads

$$i_{\delta_{susy}} \Pi^{\underline{m}} = 2\delta_{susy} X^{\underline{m}} = 2\Theta^I \sigma^{\underline{m}} \epsilon^I. \tag{4.31}$$

Imposing the boundary conditions $\hat{\Theta}^{1\underline{\mu}}(\xi(\tau)) = \hat{\Theta}^{2\underline{\mu}}(\xi(\tau))$ one arrives at the conservation of $N=1$ supersymmetry whose embedding into the type IIB supersymmetry group is defined by $\epsilon^{\underline{\mu}1} = -\epsilon^{\underline{\mu}2}$. Actually these conditions provide $i_\delta \hat{\Pi}^{\underline{m}}(\xi(\tau)) = 0$ and, as a consequence, the vanishing of the variation (4.29) (remember that the supersymmetry transformations of the gauge fields are defined by $i_\delta(\mathcal{F}_2 - F_2) = 0$).

The above consideration in the frame of the Lorentz harmonic approach results in the interesting observation that the supersymmetry breaking by a boundary is related to the 'classical reparametrization anomaly': indeed the second line of the expression (4.29), which produces the nonvanishing variation under $N=2$ supersymmetry transformation with (4.31), contains only $i_\delta \Pi^{\underline{m}}$, which can be regarded as parameters of the reparametrization gauge symmetry of the free superstring ($i_\delta \Pi^{\underline{m}} u^{\pm\pm}_{\underline{m}}$) and free super–D3–brane ($i_\delta \Pi^{\underline{m}}$), respectively.

There exists a straightforward way to keep half of the rigid target space supersymmetry of the superstring–super-D9-brane system by incorporation of the additional boundary term $\int_{\partial\mathcal{M}^{1+1}} \phi_{1\underline{\mu}} \left(\hat{\Theta}^{1\underline{\mu}}(\xi(\tau)) - \hat{\Theta}^{2\underline{\mu}}(\xi(\tau))\right)$ with a Grassmann Lagrange multiplier one form $\phi_{1\underline{\mu}}$ (see Appendix A in [22]). However, following [2, 6, 22], we accept in our present paper the 'soft' breaking of the supersymmetry by boundaries at the classical level (see [19, 2] for symmetry restoration by anomalies). We expect that the BPS states preserving part of the target space supersymmetry will appear as particular solutions of the coupled superbrane equations following from our action.

5 Current forms and unified description of string and D9-brane

5.1 Supersymmetric current form

For a simultaneous description of super–D9–brane and fundamental superstring, we have to define an 8-form distribution J_8 with support on the string worldsheet. In the pure bosonic case (see e.g. [25]) one requires

$$\int_{\mathcal{M}^{1+1}} \hat{\mathcal{L}}_2 = \int_{\underline{\mathcal{M}}^{1+9}} J_8 \wedge \mathcal{L}_2, \tag{5.1}$$

where

$$\mathcal{L}_2 = \frac{1}{2} dX^{\underline{m}} \wedge dX^{\underline{n}} \mathcal{L}_{\underline{nm}}(X^{\underline{l}}) \tag{5.2}$$

is an arbitrary two-form in the $D=10$ dimensional space-time $\underline{\mathcal{M}}^{1+9}$ and

$$\hat{\mathcal{L}}_2 = \frac{1}{2} d\hat{X}^{\underline{m}}(\xi) \wedge d\hat{X}^{\underline{n}}(\xi) \mathcal{L}_{\underline{nm}}(\hat{X}^{\underline{m}}(\xi)) \tag{5.3}$$

is its pull-back onto the string worldsheet.

It is not hard to verify that the appropriate expression for the current form J_8 is given by [25]

$$J_8 = (dX)^{\wedge 8}_{\underline{nm}} J^{\underline{nm}}(X) = \frac{1}{2!8!} \epsilon_{\underline{mnn}_1 \ldots \underline{n}_8} dX^{\underline{n}_1} \wedge \ldots \wedge dX^{\underline{n}_8} \int_{\mathcal{M}^{1+1}} d\hat{X}^{\underline{m}}(\xi) \wedge d\hat{X}^{\underline{n}}(\xi) \delta^{10}\left(X - \hat{X}(\xi)\right). \tag{5.4}$$

Indeed, inserting (5.4) and (5.2) into r.h.s. of (5.1), using (3.13) and changing the order of integrations, after performing the integration over $d^{10}X$ one arrives at the l.h.s. of (5.1).

5.2 Superstring boundaries and current (non)conservation

If the superstring worldsheet is closed ($\partial\mathcal{M}^{1+1} = 0$) the current $J^{\underline{mn}}$ is conserved, i.e. J_8 is a closed form

$$\partial\mathcal{M}^{1+1} = 0 \qquad \Rightarrow \qquad dJ_8 = 0, \qquad \Leftrightarrow \qquad \partial_{\underline{m}} J^{\underline{nm}} = 0. \tag{5.5}$$

For the open (super)string this does not hold. Indeed, assuming that the 10-dimensional space and the D9-brane worldvolume has no boundaries $\partial\mathcal{M}^{1+9} = 0$, substituting instead of $\mathcal{L}_2$ a closed two form, say dA, and using Stokes' theorem one arrives at

$$\int_{\partial\mathcal{M}^{1+1}} A = \int_{\mathcal{M}^{1+1}} dA = \int_{\mathcal{M}^{1+9}} J_8 \wedge dA = \int_{\mathcal{M}^{1+9}} dJ_8 \wedge A. \tag{5.6}$$

Thus the form dJ_8 has support localized at the boundary of the worldsheet (i.e. on the worldline of the string endpoints).

This again can be justified by an explicit calculation with Eqs. (5.4) and (3.13), which results in

$$dJ_8 = -(dX)^{\wedge 9}_{\underline{n}} \partial_{\underline{m}} J^{\underline{mn}}(X) = -(dX)^{\wedge 9}_{\underline{n}} j^{\underline{n}}(X) \tag{5.7}$$

$$\partial_{\underline{m}} J^{\underline{mn}}(X) = -j^{\underline{n}}(X),$$

with

$$j^{\underline{n}}(X) \equiv \int_{\partial\mathcal{M}^{1+1}} d\hat{X}^{\underline{m}}(\tau)\delta^{10}\left(X - \hat{X}(\tau)\right), \tag{5.8}$$

where the proper time τ parametrizes the boundary of the string worldsheet $\partial\mathcal{M}^{1+1} = \{\tau\}$.

Actually the boundary of superstring(s) shall have (at least) two connected components $\partial\mathcal{M}^{1+1} = \oplus_j \mathcal{M}^1_j$, each parametrized by the own proper time τ_j. Then the rigorous expression for the boundary current (5.7), (5.8) is

$$j^{\underline{n}}(X) \equiv \Sigma_j \int_{\mathcal{M}^1_j} d\hat{X}^{\underline{m}}(\tau_j)\delta^{10}\left(X - \hat{X}(\tau_j)\right).$$

We, however, will use the simplified notations (5.8) in what follows.

It is useful to define the local density 1–form j_1 on the worldsheet with support on the boundary of worldsheet

$$j_1 = d\xi^\mu \epsilon_{\mu\nu} \int_{\partial\mathcal{M}^{1+1}} d\tilde{\xi}^\nu(\tau)\delta^2\left(\xi - \tilde{\xi}(\tau)\right)\ , \tag{5.9}$$

which has the properties

$$\int_{\partial\mathcal{M}^{1+1}} \hat{A} \equiv \int_{\mathcal{M}^{1+1}} d\hat{A} = \int_{\mathcal{M}^{1+1}} j_1 \wedge \hat{A} = \int_{\mathcal{M}^{1+9}} dJ_8 \wedge \hat{A} \tag{5.10}$$

for any 1-form

$$A = dX^{\underline{m}} A_{\underline{m}}(X),$$

e.g., for the D9-brane gauge field (2.4) considered in the special parametrization (2.5), (2.6).

In the sense of the last equality in (5.10) one can write a formal relation

$$dJ_8 = J_8 \wedge j_1 \tag{5.11}$$

(which cannot be treated straightforwardly as the form j_1 can not be regarded as pull-back of a 10-dimensional 1-form).

5.3 Variation of current form distributions and supersymmetry

The variation of the form (5.4) becomes

$$\delta J_8 = 3(dX)^{\wedge 8}_{[\underline{mn}}\partial_{\underline{k}]} \int_{\mathcal{M}^{1+1}} d\hat{X}^{\underline{m}}(\xi) \wedge d\hat{X}^{\underline{n}}(\xi) \left(\delta X^{\underline{k}} - \delta\hat{X}^{\underline{k}}(\xi)\right)\ \delta^{10}\left(X - \hat{X}(\xi)\right)\ - \tag{5.12}$$

$$-2(dX)^{\wedge 8}_{\underline{mn}} \int_{\partial\mathcal{M}^{1+1}} d\hat{X}^{\underline{m}}(\tau) \left(\delta X^{\underline{n}} - \delta\hat{X}^{\underline{n}}(\tau)\right)\ \delta^{10}\left(X - \hat{X}(\tau)\right)\ .$$

Let us turn to the target space supersymmetry transformations (4.30). For the string coordinate fields it has the form

$$\delta\hat{X}^{\underline{m}}(\xi) = \hat{\Theta}^I(\xi)\sigma^{\underline{m}}\epsilon^I, \qquad \delta\hat{\Theta}^{I\underline{\mu}} = \epsilon^{I\underline{\mu}} \tag{5.13}$$

while for the super-D9-brane it reads

$$\delta X^{\underline{m}}(x) = \Theta^I(x)\sigma^{\underline{m}}\epsilon^I, \qquad \delta\Theta^{I\underline{\mu}}(x) = \epsilon^{I\underline{\mu}}. \tag{5.14}$$

In the parametrization (2.6) corresponding to the introduction of the inverse function (2.5) the transformation (5.14) coincides with the Goldstone fermion realization

$$\delta X^{\underline{m}} = \Theta^I(X)\sigma\epsilon^I, \qquad \delta\Theta^{I\underline{\mu}}(X) \equiv \Theta^{I\underline{\mu}\,\prime}(X') - \Theta^{I\underline{\mu}}(X) = \epsilon^{I\underline{\mu}} \tag{5.15}$$

Thus, if we identify the $X^{\underline{m}}$ entering the current density (5.4) with the bosonic coordinates of superspace, parametrizing the super-D9-brane by (2.5), we can use (5.12) to obtain the supersymmetry transformations of the current density (5.4)

$$\delta J_8 = 3(dX)^{\wedge 8}_{[\underline{mn}}\partial_{\underline{k}]} \int_{\mathcal{M}^{1+1}} d\hat{X}^{\underline{m}}(\xi) \wedge d\hat{X}^{\underline{n}}(\xi) \left(\Theta^I(X) - \hat{\Theta}^I(\xi)\right) \sigma^{\underline{k}}\epsilon^I\ \delta^{10}\left(X - \hat{X}(\xi)\right)\ - \tag{5.16}$$

$$-2(dX)^{\wedge 8}_{\underline{mn}} \int_{\partial\mathcal{M}^{1+1}} d\hat{X}^{\underline{m}}(\tau) \left(\Theta^I(X) - \hat{\Theta}^I(\tau)\right) \sigma^{\underline{n}}\epsilon^I\ \delta^{10}\left(X - \hat{X}(\tau)\right)\ .$$

Now it is evident that the current density (5.4) becomes invariant under the supersymmetry transformations (5.13), (5.14) after the identification

$$\hat{\Theta}^I(\xi) = \Theta^I\left(\hat{X}(\xi)\right) \tag{5.17}$$

of the superstring coordinate fields $\hat{\Theta}^I(\xi)$ with the image $\Theta^I\left(\hat{X}(\xi)\right)$ of the super–D9–brane coordinate field $\Theta^I(X)$ is implied.

5.4 Manifestly supersymmetric representations for the distribution form

In the presence of the D9-brane whose world volume spans the whole $D = 10$ dimensional super-time, we can rewrite (5.4) as

$$J_8 = (dx)^{\wedge 8}_{nm} J^{nm}(x) = \frac{1}{2!8!}\epsilon_{nmn_1\ldots n_8} dx^{n_1} \wedge \ldots \wedge dx^{n_8} \int_{\mathcal{M}^{1+1}} d\hat{x}^m(\xi) \wedge d\hat{x}^n(\xi)\delta^{10}\left(x - \hat{x}(\xi)\right), \tag{5.18}$$

where the function $\hat{x}(\xi)$ is defined through $\hat{X}(\xi)$ with the use of the inverse function (2.5), i.e. $\hat{X}(\xi) = X(\hat{x}(\xi))$, cf. (2.9).

Passing from (5.4) to (5.18) the identity

$$\delta^{10}\left(X - \hat{X}(\xi)\right) \equiv \delta^{10}\left(X - X(\hat{x}(\xi))\right) = \frac{1}{det(\frac{\partial X}{\partial x})}\delta^{10}\left(x - \hat{x}(\xi)\right) \tag{5.19}$$

has to be taken into account.

The consequences of this observation are two-fold:

- **i)** We can use J_8 to represent an integral over the string worldsheet as an integral over the D9-brane worldvolume [4]
$$\int_{\mathcal{M}^{1+1}} \hat{\mathcal{L}}_2 = \int_{\mathcal{M}^{1+9}} J_8 \wedge \mathcal{L}_2 \tag{5.20}$$
for any 2–form
$$\mathcal{L}_2 = \frac{1}{2} dx^m \wedge dx^n \mathcal{L}_{nm}(x^l) \tag{5.21}$$
living on the D9–brane world volume $\mathcal{M}^{1+9}$, e.g. for the field strength $\mathcal{F}_2 = dA - B_2$ (3.8) of the D9-brane gauge field (2.4). The pull–back
$$\hat{\mathcal{L}}_2 = \frac{1}{2} d\hat{x}^m(\xi) \wedge d\hat{x}^n(\xi)\mathcal{L}_{nm}(\hat{x}^l(\xi)) \tag{5.22}$$
is defined in (5.20) with the use of the inverse function (2.5).

- **ii)** As the coordinates x^n are inert under the target space supersymmetry (4.30), the current density J_8 is supersymmetric invariant. *Hence, when the identification (5.17)*
$$\hat{\Theta}^I(\xi) = \Theta^I\left(\hat{x}(\xi)\right) \tag{5.23}$$
is made, it is possible to use Eqs. (5.1), (5.20), (5.18) to lift the complete superstring action (4.28) to the 10-dimensional integral form.

The manifestly supersymmetric form of the current density appears after passing to the supersymmetric basis (2.15), (2.18) of the space tangential to $\mathcal{M}^{1+9}$. With the decomposition (2.15) J_8 becomes

$$J_8 = (\Pi)^{\wedge 8}_{\underline{nm}} J^{\underline{nm}}_{(s)}(X) = \frac{1}{2!8!}\epsilon_{\underline{nmn}_1\ldots\underline{n}_8}\Pi^{\underline{n}_1} \wedge \ldots \wedge \Pi^{\underline{n}_8} \frac{1}{det(\Pi_{\underline{r}}{}^{\underline{s}})} \int_{\mathcal{M}^{1+1}} \hat{\Pi}^{\underline{n}} \wedge \hat{\Pi}^{\underline{m}} \delta^{10}\left(X - \hat{X}(\xi)\right). \tag{5.24}$$

[4] Note the difference of the manifolds involved into the r.h.s-s of (5.20) and (5.1). This will be important for the supersymmetric case.

In Eq. (5.24) the only piece where the supersymmetric invariance is not manifest is $\delta^{10}\left(X - \hat{X}(\xi)\right)$. However, in terms of D9-brane world volume coordinates we arrive at

$$J_8 = \frac{1}{2!8!}\epsilon_{\underline{n}\underline{m}\underline{n}_1\ldots\underline{n}_8}\Pi^{\underline{n}_1}\wedge\ldots\wedge\Pi^{\underline{n}_8}\frac{1}{det(\Pi_r^{\underline{s}})}\int_{\mathcal{M}^{1+1}}\hat{\Pi}^{\underline{m}}\wedge\hat{\Pi}^{\underline{n}}\delta^{10}\left(x-\hat{x}(\xi)\right)\ , \tag{5.25}$$

where the determinant in the denominator is calculated for the matrix $\Pi_n^{\underline{m}} = \partial_n X^{\underline{m}}(x) - i\partial_n\Theta^1\sigma^{\underline{m}}\Theta^1 - i\partial_n\Theta^2\sigma^{\underline{m}}\Theta^2$. (2.17).

The manifestly supersymmetric expression for the exact dual current 9-form dJ_8 (5.7) is provided by

$$dJ_8 \equiv (dx)_n^{\wedge 9}\partial_m J^{mn}(x) = -\frac{1}{det(\Pi_r^{\underline{s}})}(\Pi)^{\wedge 9}_{\underline{m}}\int_{\partial\mathcal{M}^{1+1}}\hat{\Pi}^{\underline{m}}\delta^{10}\left(x-\hat{x}(\tau)\right). \tag{5.26}$$

$$\partial_m J^{mn}(x) = -j^n(x), \qquad j^n(x) \equiv \int_{\partial\mathcal{M}^{1+1}} d\hat{x}^n(\tau)\delta^{10}\left(x-\hat{x}(\tau)\right). \tag{5.27}$$

6 An action for the coupled system

In order to obtain a covariant action for the coupled system with the current form J_8, one more step is needed. Indeed, our lifting rules (5.1) with the density J_8 (5.4), (5.18) are valid for a form $\hat{\mathcal{L}}_2$ which is the pull-back of a form $\mathcal{L}_2$ living either on the whole $D = 10$ type IIB superspace (5.2), or, at least, on the whole 10-dimensional worldvolume of the super-D9-brane (5.21). Thus imposing the identification (5.23) we can straightforwardly rewrite the Wess-Zumino term $\int \hat{B}_2$ and the boundary term of the superstring action $\int \hat{A}$ as integrals over the super–D9–brane world volume $\int J_8 \wedge B_2 + \int dJ_8 \wedge A$.

But the 'kinetic term' of the superstring action

$$\int_{\mathcal{M}^{1+1}}\hat{\mathcal{L}}_0 \equiv \int_{\mathcal{M}^{1+1}}\frac{1}{2}\hat{E}^{++}\wedge\hat{E}^{--} \tag{6.1}$$

with $\hat{E}^{++}, \hat{E}^{--}$ defined by Eqs. (4.2) requires an additional consideration regarding the harmonics (4.3), which so far were defined only as worldsheet fields.

In order to represent the kinetic term (6.1) as an integral over the D9-brane world volume too, we have to introduce a counterpart of the harmonic fields (4.3), (4.12) *in the whole 10-dimensional space or in the D9-brane world volume*

$$u_{\underline{m}}^{\underline{a}}(x) \equiv (u_{\underline{m}}^{++}(x), u_{\underline{m}}^{--}(x), u_{\underline{m}}^{i}(x)) \qquad \in \qquad SO(1,9) \tag{6.2}$$

$$v_{\underline{\mu}}^{\underline{\alpha}} = \left(v_{\underline{\mu}q}^{I+}, v_{\underline{\mu}\dot{q}}^{I-}\right) \qquad \in \qquad Spin(1,9) \tag{6.3}$$

(see (4.3)–(2.22)).

Such a 'lifting' of the harmonics to the super–D9–brane worldvolume creates the fields of an auxiliary ten dimensional $SO(1,9)/(SO(1,1)\times SO(8))$ 'sigma model'. The only restriction for these new fields is that they should coincide with the 'stringy' harmonics on the worldsheet:

$$u_{\underline{m}}^{\underline{a}}(x(\xi)) = \hat{u}_{\underline{m}}^{\underline{a}}(\xi) \ :$$

$$u_{\underline{m}}^{++}(x(\xi)) = \hat{u}_{\underline{m}}^{++}(\xi), \qquad \hat{u}_{\underline{m}}^{--}(x(\xi)) = \hat{u}_{\underline{m}}^{--}(\xi), \qquad u_{\underline{m}}^{i}(x(\xi)) = \hat{u}_{\underline{m}}^{i}(\xi) \tag{6.4}$$

$$v_{\underline{\mu}}^{\underline{\alpha}}(x(\xi)) = \hat{v}_{\underline{\mu}}^{\underline{\alpha}}(\xi) \ :$$

$$v_{\underline{\mu}q}^{I+}(x(\xi)) = \hat{v}_{\underline{\mu}q}^{I+}(\xi), \qquad v_{\underline{\mu}\dot{q}}^{I-}(x(\xi)) = \hat{v}_{\underline{\mu}\dot{q}}^{I-}(\xi) \tag{6.5}$$

In this manner we arrive at the full supersymmetric action describing the coupled system of the open fundamental superstring interacting with the super–D9–brane (cf. (3.1)–(3.10), (4.1)):

$$\begin{aligned} S = & \int_{\mathcal{M}^{10}} (\mathcal{L}_{10} + J_8 \wedge \mathcal{L}_{IIB} + dJ_8 \wedge A) = \\ = & \int_{\mathcal{M}^{10}} \left[\Pi^{\wedge 10} \sqrt{-det(\eta_{\underline{mn}} + F_{\underline{mn}})} + Q_8 \wedge \left(dA - B_2 - \frac{1}{2} \Pi^{\underline{m}} \wedge \Pi^{\underline{n}} \, F_{\underline{mn}} \right) + e^{\mathcal{F}} \wedge C \mid_{10} \right] \\ + & \int_{\mathcal{M}^{10}} J_8 \wedge \left(\frac{1}{2} E^{++} \wedge E^{--} - B_2 \right) + \int_{\mathcal{M}^{10}} dJ_8 \wedge A \end{aligned} \tag{6.6}$$

7 Supersymmetric equations for the coupled system

7.1 Algebraic equations

The Lagrange multiplier Q_8 and auxiliary field $F_{\underline{mn}}$ are not involved into the superstring action (4.28), while the harmonics are absent in the super–D9–brane part (3.1)–(3.4) of the action (6.6). Thus we conclude that the *algebraic equations* (3.25), (3.26), (4.19) are the same as in the free models.

7.1.1 Equations obtained from varying the harmonics

Indeed, variation with respect to the harmonics (now extended to the whole $D = 10$ space time or, equivalently, to the super–D9–brane world volume (6.2)) produces the equations

$$J_8 \wedge E^i \wedge E^{\pm\pm} = 0 \qquad \Leftrightarrow \qquad J_8 \wedge E^i \equiv J_8 \wedge \Pi^{\underline{m}} u_{\underline{m}}^i = 0 \tag{7.1}$$

whose image on the worldsheet coincides with Eq. (4.19) [5]

$$\hat{E}^i \equiv \hat{\Pi}^{\underline{m}}(\xi) \hat{u}_{\underline{m}}^i(\xi) = 0 \ . \tag{7.2}$$

[5] The precise argument goes as follows: Take the integral of Eq. (7.1) with an arbitrary 10-dimensional test function $f(X)$. The integral of the forms $\hat{E}^i \wedge \hat{E}^{++}$ and $\hat{E}^i \wedge \hat{E}^{--}$ multiplied by *arbitrary* functions $f(\hat{X})$ vanishes

$$\int_{\mathcal{M}^{(1+1)}} \hat{E}^i \wedge \hat{E}^{++} f(\hat{X}) = 0, \qquad \int_{\mathcal{M}^{(1+1)}} \hat{E}^i \wedge \hat{E}^{--} f(\hat{X}) = 0.$$

From the arbitrariness of $f(\hat{X})$ then both 2-forms are identically zero on the world sheet $\hat{E}^i \wedge \hat{E}^{++} = \hat{E}^i \wedge \hat{E}^{--} = 0$. And from the independence of the pull–backs $\hat{E}^{++}$, $\hat{E}^{--}$ indeed (4.19) follows.

Now it becomes clear why the basis $E^{\underline{a}}$ (2.12)

$$E^{\underline{a}} = \left(E^{++}, E^{--}, E^{i}\right) \qquad E^{\pm\pm} = \Pi^{\underline{m}} u_{\underline{m}}^{\pm\pm}, \qquad E^{i} = \Pi^{\underline{m}} u_{\underline{m}}^{i}, \tag{7.3}$$

whose pull-back on the string worldsheet coincides with (4.4), is particularly convenient for the study of the coupled system. The dual basis $\nabla_{\underline{a}}$ (2.24) is constructed with the auxiliary moving frame variables (6.2), (6.4)

$$\nabla_{\underline{a}} = (\nabla_{++}, \nabla_{--}, \nabla_{i}) \equiv u_{\underline{a}}^{\underline{m}} \nabla_{\underline{m}}$$
$$\nabla_{++} = \frac{1}{2} u^{\underline{m}--} \nabla_{\underline{m}}, \qquad \nabla_{--} = \frac{1}{2} u^{\underline{m}++} \nabla_{\underline{m}}, \qquad \nabla_{i} = -u^{\underline{m}i} \nabla_{\underline{m}}, \tag{7.4}$$
$$\nabla_{\underline{m}} = \Pi^{-1}{}_{\underline{m}}^{\ n} \partial_{n}.$$

The decomposition of any form on the basis (2.12), (7.4) looks like

$$d\Theta^{\underline{\mu}I} = E^{\pm\pm} \nabla_{\pm\pm} \Theta^{\underline{\mu}I} + E^{i} \nabla_{i} \Theta^{\underline{\mu}I}, \tag{7.5}$$

($E^{\pm\pm}\nabla_{\pm\pm} \equiv E^{++}\nabla_{++} + E^{--}\nabla_{--}$) or

$$E^{+qI} \equiv d\Theta^{\underline{\mu}I} v_{\underline{\mu}q}^{\ +} = E^{\pm\pm} E_{\pm\pm}^{+qI} + E^{i} E_{i}^{+qI}, \tag{7.6}$$

$$E^{-\dot{q}I} \equiv d\Theta^{\underline{\mu}I} v_{\underline{\mu}\dot{q}}^{\ -} = E^{\pm\pm} E_{\pm\pm}^{-\dot{q}I} + E^{i} E_{i}^{-\dot{q}I} \tag{7.7}$$

(cf. (2.23)). Due to (7.2), only the terms proportional to E^{++}, E^{--} survive in the pull–backs of (7.5)–(7.7) on the superstring worldsheet

$$d\hat{\Theta}^{\underline{\mu}I}(\xi) = \hat{E}^{\pm\pm} \left(\nabla_{\pm\pm} \Theta^{\underline{\mu}I}\right)(x(\xi)), \tag{7.8}$$

$$\hat{E}^{+qI} \equiv d\hat{\Theta}^{\underline{\mu}I} \hat{v}_{\underline{\mu}q}^{\ +} = \hat{E}^{\pm\pm} \hat{E}_{\pm\pm}^{+qI}, \tag{7.9}$$

$$\hat{E}^{-\dot{q}I} \equiv d\hat{\Theta}^{\underline{\mu}I} \hat{v}_{\underline{\mu}\dot{q}}^{\ -} = \hat{E}^{\pm\pm} \hat{E}_{\pm\pm}^{-\dot{q}I}. \tag{7.10}$$

An alternative way to represent Eqs. (7.8), (7.9), (7.10) is provided by the use of the current density (5.18), (5.25) and the equivalent version (7.1) of Eq. (7.2)

$$J_8 \wedge d\Theta^{\underline{\mu}I} = J_8 \wedge E^{\pm\pm} \nabla_{\pm\pm} \Theta^{\underline{\mu}I}(x), \tag{7.11}$$

$$J_8 \wedge E^{+qI} \equiv J_8 \wedge d\Theta^{\underline{\mu}I} v_{\underline{\mu}q}^{\ +} = J_8 \wedge E^{\pm\pm} E_{\pm\pm}^{+qI}, \tag{7.12}$$

$$J_8 \wedge E^{-\dot{q}I} \equiv J_8 \wedge d\Theta^{\underline{\mu}I} v_{\underline{\mu}\dot{q}}^{\ -} = J_8 \wedge E^{\pm\pm} E_{\pm\pm}^{-\dot{q}I}. \tag{7.13}$$

On the other hand, one can solve Eq. (7.1) with respect to the current density. To this end we have to change the basis $\Pi^{\underline{m}} \rightarrow E^{\underline{a}} = \Pi^{\underline{a}} u_{\underline{m}}^{\underline{a}}$ (see (2.12), (6.2)) in the expression (5.25) (remember that $det(u) = 1$ due to (6.2)). Then the solution of (7.1) becomes

$$J_8 = \frac{1}{det(\Pi_r^{\ \underline{s}})} (E^{\perp})^{\wedge 8}\ \frac{1}{2} \int_{\mathcal{M}^{1+1}} \hat{E}^{++} \wedge \hat{E}^{--} \delta^{10}\left(x - \hat{x}(\xi)\right), \tag{7.14}$$

where

$$(E^{\perp})^{\wedge 8} \equiv \frac{1}{8!} \epsilon^{i_1 \ldots i_8} E^{i_1} \wedge \ldots \wedge E^{i_1} \tag{7.15}$$

is the local volume element of the space orthogonal to the worldsheet. The current form (7.14) includes an invariant on-shell superstring current

$$J_8 = (E^{\perp})^{\wedge 8} j(x), \qquad j(x) = \frac{1}{2det(\Pi_r{}^{\underline{s}})} \int_{\mathcal{M}^{1+1}} \hat{E}^{++} \wedge \hat{E}^{--} \delta^{10} \left(x - \hat{x}(\xi)\right) \ . \tag{7.16}$$

Note that it can be written with the use of the Lorentz harmonics only.

The supersymmetric covariant volume can be decomposed as well in terms of the orthogonal volume form

$$(\Pi)^{\wedge 10} \equiv (E^{\perp})^{\wedge 8} \wedge \frac{1}{2} E^{++} \wedge E^{--}. \tag{7.17}$$

7.1.2 Equations for auxiliary fields of super–D9–brane

Variation with respect to the D9-brane Lagrange multiplier Q_8 yields the identification of the auxiliary antisymmetric tensor field F with the generalized field strength $\mathcal{F}$ of the Abelian gauge field A

$$\mathcal{F}_2 \equiv dA - B_2 = F_2 \equiv \frac{1}{2} \Pi^{\underline{m}} \wedge \Pi^{\underline{n}} F_{\underline{nm}} \ . \tag{7.18}$$

On the other hand, from the variation with respect to the auxiliary antisymmetric tensor field $F_{\underline{nm}}$ one obtains the expression for the Lagrange multiplier Q_8

$$Q_8 = \Pi^{\wedge 8}_{\underline{nm}} \sqrt{|\eta + F|} (\eta + F)^{-1 \ \underline{nm}} \equiv - \frac{1}{\sqrt{|\eta + F|}} \ [(\eta + F)_{\underline{..}} \Pi^{\underline{\cdot}}]^{\wedge 8 \ \underline{nm}} F_{\underline{nm}}, \tag{7.19}$$

where $\Pi^{\wedge 8}_{\underline{nm}}$ is defined by (3.12) and (in a suggestive notation)

$$[(\eta + F)_{\underline{..}} \Pi^{\underline{\cdot}}]^{\wedge 8 \ \underline{nm}} = \frac{1}{2 \cdot 8!} \epsilon^{\underline{mnm_1 \ldots m_8}} (\eta + F)_{\underline{m_1 n_1}} \Pi^{\underline{n_1}} \wedge \ldots \wedge (\eta + F)_{\underline{m_8 n_8}} \Pi^{\underline{n_8}}. \tag{7.20}$$

The second form of (7.19) indicates that in the linearized approximation with respect to the gauge fields one obtains

$$Q_8 = - \Pi^{\wedge 8}_{\underline{nm}} F^{\underline{nm}} + \mathcal{O}(F^2) \equiv - \frac{1}{2} * F_2 + \mathcal{O}(F^2), \tag{7.21}$$

where $*$ denotes the $D = 10$ Hodge operation and $\mathcal{O}(F^2)$ includes terms of second and higher orders in the field $F_{\underline{nm}}$.

7.2 Dynamical bosonic equations: Supersymmetric Born-Infeld equations with the source.

The supersymmetric generalization of the Born-Infeld dynamical equations

$$dQ_8 + d\mathcal{L}_8^{WZ-D7} = - dJ_8 \tag{7.22}$$

follows from variation with respect to the gauge field. Here we have to take into account the expression for Q_8 (7.21), the identification of F with the gauge field strength (7.18) as well as the expression for the D7-brane Wess-Zumino term

$$d\mathcal{L}_8^{WZ-D7} = e^{\mathcal{F}} \wedge R|_9, \qquad R = \oplus_{n=0}^{5} R_{2n+1} = 2i d\Theta^{2\underline{\nu}} \wedge d\Theta^{1\underline{\mu}} \wedge \oplus_{n=0}^{4} \hat{\sigma}^{(2n+1)}_{\underline{\nu\mu}}. \tag{7.23}$$

Let us stress that, in contrast to the free Born–Infeld equation (3.27), Eq. (7.22) has a right hand side produced by the endpoints of the fundamental superstring.

Variation of the action with respect to $X^{\underline{m}}$ yields

$$\begin{aligned} & J_8 \wedge M_2^i \, u_{\underline{m}}^{\;i} + \\ & +2iJ_8 \wedge \left(E_q^{2+} \wedge E_q^{2+} u_{\underline{m}}^{--} - E_{\dot q}^{1-} \wedge E_{\dot q}^{1-} u_{\underline{m}}^{++} \right) + \\ & + \Pi^{\wedge 8}_{\underline{nm}} \sqrt{|\eta + F|} (\eta + F)^{-1\; \underline{nl}} \sigma_{\underline{l}\underline{\mu\nu}} \wedge \left(d\Theta^{2\underline{\mu}} - d\Theta^{1\underline{\rho}} h_{\underline{\rho}}^{\;\underline{\mu}} \right) \wedge \left(d\Theta^{2\underline{\nu}} - d\Theta^{1\underline{\epsilon}} h_{\underline{\epsilon}}^{\;\underline{\nu}} \right) + \\ & + dJ_8 \wedge \left(-\frac{1}{2} E^{\pm\pm} u^{\mp\mp \underline{n}} F_{\underline{nm}} + \frac{1}{2} E^{++} u_{\underline{m}}^{--} - \frac{1}{2} E^{--} u_{\underline{m}}^{++} \right) = 0. \end{aligned} \tag{7.24}$$

The first line of Eq. (7.24) contains the lifting to the super–D9–brane worldvolume of the 2-form $\hat{M}_2^i$ (4.15) which enters the l.h.s. of the free superstring bosonic equations (4.20)

$$M_2^i \equiv 1/2 E^{--} \wedge f^{++i} - 1/2 E^{++} \wedge f^{--i} + 2iE_q^{1+} \wedge \gamma^i_{q\dot q} E_{\dot q}^{1-} - 2iE_q^{2+} \wedge \gamma^i_{q\dot q} E_{\dot q}^{2-}. \tag{7.25}$$

The fourth line of Eq.(7.24) again is the new input from the boundary.

The second and third lines of Eq.(7.24) vanish identically on the surface of the free fermionic equations of the free D9-brane and of the free superstring, respectively. These are the Noether identities reflecting the diffeomorphism invariance of the free D9-brane and the free superstring actions. Hence, it is natural to postpone the discussion of Eq. (7.24) and turn to the fermionic equations for the coupled system.

7.3 Fermionic field equations

The variation with respect to Θ^2 produces the fermionic equation

$$\Pi^{\wedge 9}_{\underline{m}} \sqrt{|\eta + F|} (\eta + F)^{-1\; \underline{mn}} \sigma_{\underline{n}\underline{\mu\nu}} \wedge \left(d\Theta^{2\underline{\nu}} - d\Theta^{1\underline{\rho}} h_{\underline{\rho}}^{\;\underline{\nu}} \right) = -2J_8 \wedge E^{--} \wedge d\Theta^{2\underline{\nu}} \, v_{\underline{\nu}q}^{\;+} \, v_{\underline{\mu}q}^{\;+}, \tag{7.26}$$

with the r.h.s. localized at the worldsheet and proportional to the l.h.s. of the fermionic equations of the free type IIB superstring (cf. (4.22) and remembering that $d\Theta^{2\underline{\nu}} \, v_{\underline{\nu}q}^{\;+} \equiv E_q^{2+}$).

The remaining fermionic variation $\delta\Theta^1$ produces an equation which includes the form J_8 with support localized at the worldsheet only:

$$J_8 \wedge \left(E^{++} \wedge E_{\dot q}^{1-} v_{\underline{\mu}\dot q}^{\;-} - E^{--} \wedge E_q^{2+} h_{\underline{\mu}}^{\;\underline{\nu}} v_{\underline{\nu}q}^{\;+} \right) = 0. \tag{7.27}$$

This equation is worth a special consideration. For clearness, let us write its image on the worldsheet

$$\hat{E}^{++} \wedge \hat{E}_{\dot q}^{1-} \hat{v}_{\underline{\mu}\dot q}^{\;-} - \hat{E}^{--} \wedge \hat{E}_q^{2+} \hat{h}_{\underline{\mu}}^{\;\underline{\nu}} \hat{v}_{\underline{\nu}q}^{\;+} = 0. \tag{7.28}$$

Contracting with inverse harmonics $v_q^{-\underline{\mu}}, v_{\dot q}^{+\underline{\mu}}$ and using the 'multiplication table' of the harmonics ((A.7)) we arrive at the following covariant $8+8$ splitting representation for the 16 equations (7.28):

$$\hat{E}^{++} \wedge \hat{E}_{\dot q}^{1-} = \hat{E}^{--} \wedge \hat{E}_q^{2+} \hat{h}_{\dot q q}^{++} \tag{7.29}$$

$$\hat{E}^{--} \wedge \hat{E}_p^{2+} \hat{h}_{qp} = 0. \tag{7.30}$$

Here

$$\hat{h}^{++}_{\dot{q}q} \equiv \hat{v}^{+\mu}_{\dot{q}}{}^{-}\hat{h}_{\underline{\mu}}{}^{\underline{\nu}}\hat{v}_{\underline{\nu}q}{}^{+}, \tag{7.31}$$

$$\hat{h}_{pq} \equiv \hat{v}_p{}^{-\mu}{}^{-}\hat{h}_{\underline{\mu}}{}^{\underline{\nu}}\hat{v}_{\underline{\nu}q}{}^{+} \tag{7.32}$$

are the covariant 8 blocks of the image $\hat{h}$ of the Lorentz group valued (and hence invertible!) spin-tensor field h (3.19)-(3.21)

$$h_{\underline{\beta}}{}^{\underline{\alpha}} \equiv v_{\underline{\beta}}{}^{\underline{\nu}} h_{\underline{\mu}}{}^{\underline{\nu}} v_{\underline{\mu}}{}^{\underline{\alpha}} \equiv \begin{pmatrix} h_{qp} & h^{--}_{q\dot{p}} \\ h^{++}_{\dot{q}p} & \tilde{h}_{\dot{q}\dot{p}} \end{pmatrix}. \tag{7.33}$$

Note that the source localized on the worldsheet of the open brane, as in (7.26) is characteristic for the system including a space-time filling brane. For the structure of the fermionic equations in the general case we refer to [22].

8 Phases of the coupled system

It is useful to start with the fermionic equations of motion (7.26), (7.29), (7.30).

First of all we have to note that in the generic phase *there are no true (complete) Noether identities for the κ–symmetry* in the equations for the coupled system, as all the 32 fermionic equations are independent.

8.1 Generic phase describing decoupled system and appearance of other phases

In the generic case we shall assume that the matrix $h_{qp}(X(\xi)) = \hat{h}_{qp}(\xi)$ is invertible ($det(\hat{h}_{qp}) \neq 0$, for the case $det(\hat{h}_{qp}) = 0$ see Section 7.3). Then Eq. (7.30) implies $\hat{E}^{--} \wedge \hat{E}^{2+}_{p} = 0$ and immediately results in the reduction of the Eq. (7.29):

$$det(\hat{h}_{qp}) \neq 0 \qquad \Rightarrow \qquad \begin{cases} \hat{E}^{++} \wedge \hat{E}^{1-}_{\dot{q}} = 0 \\ \hat{E}^{--} \wedge \hat{E}^{2+}_{q} = 0 \end{cases} \tag{8.1}$$

The equations (8.1) have the same form as the free superstring equations of motion (4.21), (4.22). As a result, the r.h.s. of Eq. (7.26) vanishes

$$det(\hat{h}_{qp}) \neq 0 \qquad \Rightarrow$$

$$\Pi^{\wedge 9}_{\underline{m}} \sqrt{|\eta + F|}(\eta + F)^{-1\ \underline{mn}} \sigma_{\underline{n}\underline{\mu}\underline{\nu}} \wedge \left(d\Theta^{2\underline{\nu}} - d\Theta^{1\underline{\rho}} h_{\underline{\rho}}{}^{\underline{\nu}} \right) = 0 \tag{8.2}$$

which coincides with the fermionic equation for the free super–D9–brane (3.28). Then the third line in the equations of motion for $X^{\underline{m}}$ coordinate fields (7.24) vanishes, as it does in the free super–D9–brane case (3.32). As the second line in Eq. (7.24) is zero due to the equations (8.1) (e.g. $\hat{E}^{2+}_{q} \wedge \hat{E}^{2+}_{q} = (\hat{E}^{\pm\pm}\hat{E}_{\pm\pm q}{}^{2+}) \wedge (\hat{E}^{\mp\mp}\hat{E}_{\mp\mp q}{}^{2+}) = -2\hat{E}^{--} \wedge \hat{E}^{2+}_{q}\ \hat{E}_{--q}{}^{2+} = 0$), in the generic case (8.1) the equations of motion for X field (7.24) become

$$det(\hat{h}_{qp}) \neq 0 \qquad \Rightarrow$$

$$J_8 \wedge M_2^i \, u_{\underline{m}}^{\ i} + dJ_8 \wedge \left(-\frac{1}{2} E^{\pm\pm} u^{\mp\mp \underline{n}} F_{\underline{nm}} + \frac{1}{2} E^{++} u_{\underline{m}}^{--} - \frac{1}{2} E^{--} u_{\underline{m}}^{++} \right) = 0. \tag{8.3}$$

Contracting equation (8.3) with appropriate harmonics (6.2), one can split it into three covariant equations

$$J_8 \wedge M_2^i = \frac{1}{2} dJ_8 \wedge E^{\pm\pm} F^{\mp\mp i}, \tag{8.4}$$

$$dJ_8 \wedge E^{++} \left(1 - \frac{1}{2} F^{++\;--} \right) = 0, \tag{8.5}$$

$$dJ_8 \wedge E^{--} \left(1 - \frac{1}{2} F^{++\;--} \right) = 0, \tag{8.6}$$

where

$$F^{\mp\mp i} \equiv u^{\underline{m}\pm\pm} u^{\underline{n}i} F_{\underline{mn}}, \qquad F^{++\;--} \equiv u^{\underline{m}++} u^{\underline{n}--} F_{\underline{mn}} \tag{8.7}$$

are contractions of the antisymmetric tensor field (gauge field strength) with the harmonics (6.2).

The l.h.s. of the first equation (8.4) has support on the string world volume $\mathcal{M}^{1+1}$, while its r.h.s and all the equations (8.5), (8.6) have support on the boundary of the string worldsheet $\partial\mathcal{M}^{1+1}$ only.

An important observation is that the requirement for the superstring to have a nontrivial boundary $\partial\mathcal{M}^{1+1} \neq 0$ *implies a specific restriction for the image of the gauge field strength on the boundary of the string worldsheet*

$$\partial\mathcal{M}^{1+1} \neq 0 \qquad \Rightarrow \qquad \hat{F}^{++\;--}|_{\partial\mathcal{M}^{1+1}} \equiv \hat{u}^{\underline{m}++} \hat{u}^{\underline{n}--} F_{\underline{mn}}|_{\partial\mathcal{M}^{1+1}} = 2. \tag{8.8}$$

Eqs. (8.8) can be regarded as 'boundary conditions' for the super–D9–brane gauge fields on 1-dimensional defects provided by the endpoints of the fundamental superstring. Such boundary conditions describe a phase of the coupled system where the open superstring interacts with the D9-brane gauge fields through its endpoints.

However, the most general phase, which implies no restrictions (8.8) on the image of the gauge field, is characterized by equations $dJ_8 \wedge E^{--} = 0$, $dJ_8 \wedge E^{++} = 0$ and $dJ_8 = 0$. This means the conservation of the superstring current and thus implies that the superstring is closed

$$\hat{F}^{++\;--}|_{\partial\mathcal{M}^{1+1}} \neq 2 \quad \Rightarrow \quad dJ_8 = 0 \quad \Rightarrow \quad \partial\mathcal{M}^{1+1} = 0. \tag{8.9}$$

The equations decouple and become the equations of the free D9–brane and the ones of the free closed type IIB superstring.

Hence to arrive at the equations of a nontrivially coupled system of super–D9–brane and open fundamental superstring we have to consider phases related to special 'boundary conditions' for the gauge fields on the string worldvolume or its boundary. The weakest form of such boundary conditions are provided by (8.8).

Below we will describe some interesting phases characterized by the boundary conditions formulated on the whole superstring worldsheet, but before that some comments on the issues of κ–symmetry and supersymmetry seem to be important.

8.2 Issues of κ–symmetry and supersymmetry

8.2.1 On κ–symmetry

If one considers the field variation of the form (3.30), (3.31) for the free D9-brane κ-symmetry transformation, one finds that they describe a gauge symmetry of the coupled system as well, if the parameter κ is restricted by 'boundary conditions' on the two dimensional defect (superstring worldsheet)

$$\kappa^{\underline{\mu}}(x(\xi)) \equiv \hat{\kappa}^{\underline{\mu}}(\xi) = 0. \tag{8.10}$$

Thus we have a counterpart of the κ-symmetry inherent to the host brane (D9-brane) in the coupled system. As the defect (string worldsheet) is a subset of measure zero in 10-dimensional space (D9-brane world volume) we still can use this restricted κ–symmetry to remove half of the degrees of freedom of the fermionic fields all over the D9-brane worldvolume except for the defect.

At the level of Noether identities this 'restricted' κ–symmetry is reflected by the fact that the half of the fermionic equations (7.27) has nonzero support on the worldsheet only.

For a system of low-dimensional intersecting branes and open branes ending on branes, which does not include the super–D9–brane or other space–time filling brane we will encounter an analogous situation where the κ–symmetries related to both branes should hold outside the intersection.

However, we should note that, in the generic case (8.1), all the 32 variations of the Grassmann coordinates result in nontrivial equations. Thus we have *no true counterpart* of the free brane κ-symmetry. Let us recall that the latter results in the dependence of half of the fermionic equations of the free superbrane. It is usually identified with the part (one-half) of target space supersymmetry preserved by the BPS state describing the brane (e.g. as the solitonic solutions of the supergravity theory).

8.2.2 Bosonic and fermionic degrees of freedom and supersymmetry of the decoupled phase

As the general phase of our coupled system (8.1), (8.9) describes the decoupled super–D9–brane and closed type IIB superstring, it must exhibit the complete $D = 10$ type IIB supersymmetry. Supersymmetry (in a system with dimension $d > 1$) requires the coincidence of the numbers of bosonic and fermionic degrees of freedom. We find it instructive to consider how such a coincidence can be verified starting from the action of the coupled system, and to compare the verification with the one for the case of free branes.

In the free super–D9–brane case the 32 fermionic fields $\Theta^{\underline{\mu}I}$ can be split into 16 physical and 16 unphysical (pure gauge) ones. For our choice of the sign of the Wess-Zumino term (3.7) they can be identified with $\Theta^{\underline{\mu}2}$ and $\Theta^{\underline{\mu}1}$, respectively.

Then one can consider the equations of motion (3.28) as restrictions of the physical degrees of freedom (collected in $\Theta^{\underline{\mu}2}$), while the pure gauge degrees of freedom ($\Theta^{\underline{\mu}1}$) can be removed completely by κ–symmetry (3.30), i.e. we can fix a gauge $\Theta^{\underline{\mu}1} = 0$ (see [40]).

A similar situation appears when one considers the free superstring model, where one can identify the physical degrees of freedom with the set of $\hat{\Theta}^{1-}_{\dot{q}} = \hat{\Theta}^{\underline{\mu}1}\hat{v}^{-}_{\underline{\mu}\dot{q}}, \hat{\Theta}^{2+}_{q} = \hat{\Theta}^{\underline{\mu}2}\hat{v}^{+}_{\underline{\mu}q}$ (4.25), while the remaining components $\hat{\Theta}^{1+}_{q} = \hat{\Theta}^{\underline{\mu}1}\hat{v}^{+}_{\underline{\mu}q}$, $\hat{\Theta}^{2-}_{\dot{q}} = \hat{\Theta}^{\underline{\mu}2}\hat{v}^{-}_{\underline{\mu}\dot{q}}$, are pure gauge degrees of freedom with respect to the κ-symmetry whose irreducible form is given by (4.16) (see (4.24)).

To calculate the number of degrees of freedom we have to remember that

- pure gauge degrees of freedom are removed from the consideration *completely*,
- the solution of second order equations of motion (appearing as a rule for bosonic fields, i.e. $\hat{X}^i(\xi) = \hat{X}^i(\tau,\sigma)$ (4.25)) for n physical variables (extracted e.g. by fixing all the gauges) is characterized by $2n$ independent functions, which can be regarded as initial data for coordinates ($\hat{X}^i(0,\sigma)$) and momenta (or velocities $\partial_\tau \hat{X}^i(0,\sigma)$),
- the general solution of the first order equations (appearing as a rule for fermions, e.g. $\hat{\Theta}^{1-}_{\dot{q}}(\tau,\sigma)$, $\hat{\Theta}^{2+}_{q}(\tau,\sigma)$) is characterized by only n functions, which can be identified with the initial data for coordinates ($\hat{\Theta}^{1-}_{\dot{q}}(0,\sigma)$, $\hat{\Theta}^{2+}_{q}(0,\sigma)$) which are identical to their momenta in this case.

In this sense it is usually stated that n physical (non pure gauge) fields satisfying the second order equations of motion *carry n degrees of freedom* (e.g. for $\hat{X}^i(\xi) \quad n = (D-2) = 8$), while n physical fields satisfying the first order equations of motion *carry $n/2$ degrees of freedom* (e.g. for $\hat{\Theta}^{1-}_{\dot{q}}(\tau,\sigma)$, $\hat{\Theta}^{2+}_{q}(\tau,\sigma) \quad n/2 = 2(D-2)/2 = 16/2 = 8$). This provides us with the same value 8 for the number of bosonic and fermionic degrees of freedom for both the free super–D9–brane and the free type II superstring ($8_B + 8_F$).

If one starts from the action of a coupled system similar to (6.6), the counting should be performed in a slightly different manner, because, as it was discussed above, *we have no true κ–symmetry* in the general case. We still count 8 physical bosonic degrees of freedom related to the super–D9–brane gauge field $A_{\underline{m}}(x)$ living in the whole bulk (super–D9–brane worldvolume), and 8 physical bosonic degrees of freedom living on the 'defect' (superstring worldsheet) related to the orthogonal oscillations of the string $\hat{X}^i(\xi)$.

The 32 fermionic coordinate fields $\Theta^{\underline{\mu}1}(x), \Theta^{\underline{\mu}2}(x)$ are restricted here by two sets of 16 equations (7.26), (7.27), with one set (7.26) involving the fields in the bulk (and also the source term with support on the worldsheet) and the other (7.27) with support on the worldsheet only.

As the field theoretical degrees of freedom are related to the general solution of homogeneous equations (in the light of the correspondence with the initial data described above), the presence (or absence) of the source with local support in the right hand part of the coupled equations is inessential and we can, in analogy with the free D9-brane case, treat the first equation (7.26) as the restriction on 16 physical fermionic fields (say $\Theta^{\underline{\mu}2}(x)$) *in the bulk*. As mentioned above, the coupled system has a D9-brane-like κ–symmetry with the parameter $\kappa^{\underline{\mu}}(x)$ restricted by the requirement that it should vanish on the defect $\hat{\kappa}^{\underline{\mu}}(\xi) \equiv \kappa^{\underline{\mu}}(x(\xi)) = 0$. Thus we can use this kappa symmetry to remove the rest of the 16 fermionic fields (say $\Theta^{\underline{\mu}1}(x)$) all over the bulk except at the defect.

Thus all over the bulk *including* the defect we have 8 bosonic fields, which are the components of $A_{\underline{m}}(x)$ modulo gauge symmetries and $8 = 16/2$ fermionic fields, which can be identified with the on-shell content of $\Theta^{\underline{\mu}2}(x)$.

On the defect we have in addition the 16 components $\hat{\Theta}^{\underline{\mu}1}(\xi)$, which are restricted (in the general case) by 16 first order equations (8.1) (or (4.21), (4.22)) and, thus, carry 8 degrees of freedom. This is the same number of degrees of freedom as the one of the orthogonal bosonic oscillations of superstring $\hat{X}^i(\xi) = \hat{X}^{\underline{m}}(\xi)\hat{u}^i_{\underline{m}}(\xi)$. This explains why our approach to the coupled system allows to describe a decoupled supersymmetric phase.

It should be remembered (see Section 3.4) that the presence of boundaries breaks at least half of the target space supersymmetry.

8.3 Phases implying restrictions on the gauge fields

As mentioned in Section 8.1, the open fundamental superstring can be described only when some restrictions on the image of the gauge field are implied. The simplest restriction is given by Eq. (8.8). But it is possible to consider the phases where (8.8) appears as a consequence of stronger restrictions which hold on the whole defect (string worldvolume), but not only on its boundary.

An interesting property of such phases is that there an interdependence of the fermionic equations of motion emerges. Such a dependence can be regarded as an additional 'weak' counterpart of the κ-symmetry of the free superbrane actions.

8.3.1 Phases with less than 8 dependent fermionic equations

The dependence of fermionic equations arises naturally when the matrix $\hat{h}_{pq}$ is degenerate:

$$det(\hat{h}_{qp}) = 0 \qquad \Leftrightarrow \qquad r_h \equiv rank(\hat{h}_{qp}) < 8 \tag{8.11}$$

Then $\hat{h}_{qp}$ may be represented through a set of $8 \times r_h$ rectangular matrices $S_q{}^I$

$$\hat{h}_{qp} = (\pm) S_q{}^I S_p{}^I, \qquad q, p = 1, \ldots 8, \qquad I = 1, \ldots r_h, \qquad r_h < 8, \tag{8.12}$$

and Eq. (7.30) implies only $r_h < 8$ nontrivial relations

$$0 < r_h < 8 \qquad \Leftrightarrow \qquad \hat{E}^{--} \wedge \hat{E}^{2+}_q S_q{}^I = 0, \qquad I = 1, \ldots r_h, \tag{8.13}$$

while the remaining $8 - r_h$ fermionic equations are dependent.

The general solution of Eq. (8.13) differs from the expression for the fermionic equations of the free superstring $\hat{E}^{--} \wedge \hat{E}^{2+}_q = 0$ by the presence of $(8 - r_h)$ arbitrary fermionic two-forms (actually functions, as on the worldsheet any two-form is proportional to the volume $\hat{E}^{++} \wedge \hat{E}^{--}$)

$$0 < r_h < 8 \qquad \Leftrightarrow \qquad \hat{E}^{--} \wedge \hat{E}^{2+}_q = R_q{}^{\tilde{J}} \hat{\Psi}_{2+}{}^{\tilde{J}} \qquad I = 1, \ldots r_h \tag{8.14}$$

where the $8 \times (8 - r_h)$ matrix $R_q{}^{\tilde{J}}$ is composed of $8 - r_h$ $SO(8)$'s–vectors' which complete the set of r_h $SO(8)$ s–vectors $S_q{}^J$ to the complete basis in the 8 dimensional space, i.e.

$$R_q{}^{\tilde{J}} S_q{}^I = 0. \tag{8.15}$$

On the other hand, due to Eqs. (8.14), (8.12), the ${R_q}^{\tilde{J}}$ are the 'null-vectors' of the matrix h_{pq}

$$\hat{h}_{qp} {R_p}^{\tilde{J}} = 0 \tag{8.16}$$

Thus, they may be used to write down the explicit form of the $8 - r_h$ dependent fermionic equations

$$rank(\hat{h}_{qp}) = r_h < 8 \qquad \Rightarrow \qquad \left(\hat{E}^{--} \wedge \hat{E}^{2+}_{q} \right) \hat{h}_{qp} {R_p}^{\tilde{J}} \equiv 0. \tag{8.17}$$

8.3.2 Nonperturbative phase with 8 dependent fermionic equations.

The case with the maximal number 8 of dependent fermionic equations appears when the matrix h_{qp} vanishes at the defect ($\hat{h}_{qp} = 0$). As the complete matrix $h_{\underline{\alpha}}{}^{\underline{\beta}}$ (7.33), (3.20), (3.21) is Lorentz group valued (3.19) and, hence, nondegenerate ($det(h_{\underline{\alpha}}{}^{\underline{\beta}}) \neq 0$), this implies that both antidiagonal 8×8 blocks $h^{--}_{q\dot{p}}$, $h^{++}_{\dot{q}p}$ are nondegenerate

$$\hat{h}_{qp} = 0, \qquad \Rightarrow \qquad det(\hat{h}^{--}_{q\dot{p}}) \neq 0, \qquad det(\hat{h}^{++}_{\dot{q}p}) \neq 0. \tag{8.18}$$

In this case the fermionic equations (7.29) are satisfied identically and thus we arrive at the system of $16 + 8 = 24$ nontrivial fermionic equations. The dependence of Eq. (7.29) for the gauge field subject to the 'boundary conditions' (8.18) (see (3.20), (3.21)) can be regarded as a counterpart of 8 κ–symmetries. Thus it could be expected that ground state solutions corresponding to the BPS states preserving $1/4$ (i.e. 8) of the 32 target space supersymmetries should appear just in this phase.

It is important that the phase (8.18) is *nonperturbative* in the sense that it has no a weak gauge field limit. Indeed, in the limit $F_{\underline{mn}} \to 0$ the spin-tensor $h_{\underline{\nu}}{}^{\underline{\mu}}$ (3.19), (3.20) should tend to unity $h_{\underline{\nu}}{}^{\underline{\mu}} = \delta_{\underline{\nu}}{}^{\underline{\mu}} + \mathcal{O}(F)$. As $\hat{v}_p^{-\underline{\mu}} \hat{v}^{+}_{\underline{\mu}q} = \delta_{pq}$ (see Appendix A), the same is true for the $SO(8)$ s–tensor h_{pq}: $h_{pq} = \delta_{pq} + \mathcal{O}(F)$. Thus the condition (8.18) cannot be obtained in the weak field limit. This reflects the fact that nontrivial coupling of the gauge field with string endpoints is described by this phase.

Another way to justify the above statements is to use (3.20), (3.21) with the triangle matrix (7.33)

$$\hat{h}_{\underline{\beta}}{}^{\underline{\alpha}} \equiv \hat{v}_{\underline{\beta}}{}^{\underline{\nu}} \hat{h}_{\underline{\mu}}{}^{\underline{\nu}} \hat{h}_{\underline{\mu}}{}^{\underline{\alpha}} \equiv \begin{pmatrix} 0 & \hat{h}^{--}_{q\dot{p}} \\ \hat{h}^{++}_{\dot{q}p} & \tilde{\hat{h}}_{\dot{q}\dot{p}} \end{pmatrix} \tag{8.19}$$

and the explicit $SO(1,1) \times SO(8)$ invariant representation for σ–matrices (see Eq. (A.8) in the Appendix A) to find that $\hat{h}_{pq} = 0$ implies (see Appendix C)

$$\hat{F}^{++\,--} \equiv \hat{u}^{\underline{m}++} \hat{u}^{\underline{n}--} \hat{F}_{\underline{mn}} = 2. \tag{8.20}$$

Thus we see again that there is no weak gauge field limit, as the image of at least one of the gauge field strength components onto the string worldsheet has a finite value in the phase (8.18). On the other hand, Eq. (8.20) demonstrates that the condition (8.8) holds on the boundary of the worldsheet. Thus one can expect that this phase provides a natural possibility to describe the nontrivial coupling of the *open* fundamental superstring with the D-brane gauge field. As we will prove below analysing the field equations, this is indeed the case.

In the 'nonperturbative phase' (8.18) one of the fermionic equations (7.30) is satisfied identically and thus we have only one nontrivial fermionic equation (7.29) on the string worldsheet. Using the consequences (7.9) of Eq. (7.2) the 2-form equation (7.29) can be decomposed as

$$\hat{E}^{++} \wedge \hat{E}^{--} (\hat{E}^{1}{}_{--\dot{q}}^{-} + \hat{E}^{2}{}_{++q}^{+} \hat{h}^{++}_{\dot{q}q}) = 0. \tag{8.21}$$

We find that it contains eight 0-form fermionic equations

$$\hat{E}^{1}{}_{--\dot{q}}^{-} = -\hat{E}^{2}{}_{++q}^{+} \hat{h}^{++}_{\dot{q}q}. \tag{8.22}$$

Another version of Eq. (8.22) is

$$J_8 \quad \left(E^{1}{}_{--\dot{q}}^{-} + E^{2}{}_{++q}^{+} h^{++}_{\dot{q}q} \right) = 0. \tag{8.23}$$

In the linearized approximation with respect to all fields *except for the gauge field strength F* ($h^{++}_{\dot{q}q} = \mathcal{O}(F)$) the equation (8.22) becomes

$$\partial_{--} \hat{\Theta}^{-1}_{\dot{q}} = -\partial_{++} \hat{\Theta}^{+2}_{q} \hat{h}^{++}_{\dot{q}q}. \tag{8.24}$$

One should remember that the free superstring fermionic equations (4.21), (4.22) as well as the equations (7.29), (7.30) in the generic phase (8.1) imply $det(\hat{h}_{qp}) \neq 0, \quad \Rightarrow \quad \hat{E}^{1}{}_{--\dot{q}}^{-} = 0, \; \hat{E}^{2}{}_{++q}^{+} = 0$, whose linearized limit is $\partial_{--} \hat{\Theta}^{-1}_{\dot{q}} = 0, \; \partial_{++} \hat{\Theta}^{+2}_{q} = 0$ with chiral fields as a solution $\hat{\Theta}^{-1}_{\dot{q}} = \hat{\Theta}^{-1}_{\dot{q}}(\xi^{(++)}), \; \hat{\Theta}^{+2}_{q} = \hat{\Theta}^{+2}_{q}(\xi^{(--)})$.

The rest of the fermionic equations (7.26)

$$\begin{aligned} \Pi^{\wedge 9}_{\underline{m}} \sqrt{|\eta + F|} (\eta + F)^{-1\ \underline{mn}} \sigma_{\underline{n}\underline{\mu}\underline{\nu}} \wedge \left(d\Theta^{2\underline{\nu}} - d\Theta^{1\underline{\rho}} h_{\underline{\rho}}{}^{\underline{\nu}} \right) &= -2J_8 \wedge E^{--} \wedge E^{2+}_{q} v^{\ +}_{\underline{\mu}q} \\ &\equiv -2J_8 \wedge E^{++} \wedge E^{1-}_{\dot{q}} (h^{++})^{-1}_{q\dot{q}} v^{\ +}_{\underline{\mu}q} \end{aligned} \tag{8.25}$$

has a nontrivial source localized on the superstring worldsheet. It is just proportional to the expression which vanishes in the free superstring case and in the generic phase, but remains nonzero in the phase (8.18).

In the present case the relations (8.8) hold (see Eq. (8.20)). Using these relations, straightforward but tedious calculations demonstrate that the projections of Eqs. (7.24) for $\delta X^{\underline{m}}$ onto the harmonics $u^{\pm\pm}_{\underline{m}}$ vanish identically here (these are Noether identities for reparametrization symmetry on the superstring worldvolume), while the projection onto $u^{\ i}_{\underline{m}}$ results in

$$J_8 \wedge M^i_2 = -\frac{1}{2} dJ_8 \wedge (E^{++} F^{--i} + E^{--} F^{++i}), \tag{8.26}$$

where F^{++i}, F^{--i} and M^i_2 are defined in Eqs. (8.7) and (7.25) respectively. Eq. (8.26) differs from the one of the free superstring by the nonvanishing r.h.s., which has support *on the boundary* of the string worldsheet and describes the interaction with super-D9-brane gauge fields.

The Born-Infeld equations has the form (7.22) (with (7.19), (7.18) taken into account) and contains a nonvanishing source term $-dJ_8$.

Thus, as expected, the phase (8.18) describes the open fundamental superstring interacting with the super-D9-brane. The ends of the superstring carry the charge of the super-D9-brane gauge field and provide the source for the supersymmetric Born-Infeld equation. Note that the source of the fermionic equations is localized on the whole worldsheet. This property is specific for the system including a space–time filling brane.

9 Conclusion and outlook

In this paper we present the derivation of a complete set of supersymmetric equations for a coupled system consisting of the super–D9–brane and the open fundamental superstring 'ending' on the D9-brane. To this end we construct a current distribution form J_8 which allows to write the action functional of superstring and D9-brane in similar forms, i.e. as an integral of a 10-form over the 10-dimensional space, after the Grassmann coordinates of the superstring are identified with the images of the Grassmann coordinate fields of the super–D9–brane. We prove supersymmetric invariance of J_8.

The proposed way to construct the action for the coupled system of superstring and space–time filling brane requires the use of the moving frame (Lorentz harmonic) actions [31, 14, 15] for the superstring. The reason is that its Lagrangian form (in distinction to the ones of the standard action [20]) can be regarded as pull–backs of some D-dimensional differential 2–form and, thus, the moving frame actions for the free superstring can be written easily as an integral over a D–dimensional manifold by means of the current density J_8. Just the existence of the moving frame formulation may motivate the *formal* lifting of the Lagrangian forms of the standard actions to D dimensions and their use for the description of the interaction with space–time filling branes and/or supergravity (see [25] for bosonic branes).

We obtain a complete supersymmetric system of the equations of motion for the coupled system of superstring and super–D9–brane. Different phases of the coupled system are found. One of them can be regarded as generic, but describes the decoupled system of the closed superstring and the super-D9-brane, while one of the others corresponds to a 'singular' and nonperturbative 'boundary condition' for the gauge field on the worldsheet. It describes the coupled system of the *open* superstring interacting with the D9-branes and implies an interdependence of the fermionic equations of motion which can be regarded as a weak counterpart of the (additional) κ–symmetry.

The method proposed in [22] and elaborated in this paper may also be applied to the construction of the action for a coupled system containing any number N_2 of fundamental superstrings and any number N_{2k} of type IIB super-Dp-branes ($p = 2k - 1$) interacting with the super-D9-brane. In the action of such a coupled system

$$S = \int_{\mathcal{M}^{1+9}} \left(\mathcal{L}_{10} + \sum_{k=1}^{4} \sum_{r_{2k}=1}^{N_{p=2k}} \int_{\mathcal{M}^{1+9}} J^{(r_{2k})}_{10-2k} \wedge \mathcal{L}^{(r_{2k})}_{2k} \right) + \tag{9.1}$$

$$+ \sum_{s=1}^{N_2} \int_{\mathcal{M}^{1+9}} J^{(s)}_8 \wedge \left(\mathcal{L}^{(s)}_2 + dA \right)$$

$\mathcal{L}_{10} = \mathcal{L}^0_{10} + \mathcal{L}^1_{10} + \mathcal{L}^{WZ}_{10}$ is the Lagrangian form of the super-D9-brane action (3.1)–(3.4), (3.7). $\mathcal{L}^{(s)}_2$ represents the Lagrangian form (4.1) for the s-th fundamental superstring lifted to the 9-brane world volume as in (6.6), $J^{(s)}_8$ is the local supersymmetric current density (5.18), (5.25) for the s-th fundamental superstring. The latter is constructed with the help of the induced map of the worldsheet into the 10-dimensional worldvolume of the super–D9–brane. Finally, $\mathcal{L}^{(r)}_{2k}$ and $J^{(r)}_{10-2k}$ are the supersymmetric current density and a first order action functional for the r-th type IIB super-Dp-brane with $p = 2k-1 = 1,3,5,7$. The supersymmetric current density $J^{(r)}_{10-2k}$

$$
\begin{aligned}
J^{(r)}_{10-2k} &= (dx)^{\wedge 10-2k}_{n_1 \ldots n_{2k}} J^{n_1 \ldots n_{2k}}(x) = \\
&= \frac{1}{(10-2k)!(2k)!} \epsilon_{m_1 \ldots m_{10-2k} n_1 \ldots n_{2k}} dx^{m_1} \wedge \ldots \wedge dx^{m_{10-2k}} \times \\
&\times \int_{\mathcal{M}^{1+2k}} d\hat{x}^{(r)n_1}(\zeta) \wedge \ldots \wedge d\hat{x}^{(r)n_{2k}}(\xi) \delta^{10}\left(x - \hat{x}^{(r)}(\xi)\right) \equiv \\
&= \frac{1}{(10-2k)!(2k)!} \epsilon_{\underline{m}_1 \ldots \underline{m}_{10-2k} \underline{n}_1 \ldots \underline{n}_{2k}} \frac{\Pi^{\underline{m}_1} \wedge \ldots \wedge \Pi^{\underline{m}_{10-2k}}}{det(\Pi_l{}^{\underline{k}})} \times \\
&\times \int_{\mathcal{M}^{1+1}} \hat{\Pi}^{(r)\underline{n}_1}(\zeta) \wedge \ldots \wedge \hat{\Pi}^{(r)\underline{n}_{2k}}(\zeta) \delta^{10}\left(x - \hat{x}^{(r)}(\zeta)\right)
\end{aligned}
\tag{9.2}
$$

is defined by the induced map $x^m = \hat{x}^{(r)m}(\zeta)$ $(m = 0, \ldots 9)$ of the r-th Dp-brane worldvolume into the 10-dimensional worldvolume of the D9-brane, given by

$$
\hat{X}^{(r)\underline{m}}(\zeta) = X^{\underline{m}}\left(\hat{x}^{(r)}(\zeta)\right) \qquad \leftrightarrow \qquad \hat{x}^{(r)m}(\zeta) = x^m\left(\hat{X}^{(r)\underline{m}}(\zeta)\right). \tag{9.3}
$$

The Lagrangian form $\mathcal{L}_{2k}$ of the first order action for the free super-Dp-brane with $p = 2k$ can be found in [32]. Certainly the form of the interaction between branes, which can be introduced into $\mathcal{L}^{(r)}_{2k}$ by the boundary terms requires a separate consideration (e.g. one of the important points is the interaction with the D9-brane gauge field through the Wess-Zumino terms of Dp-branes). We hope to return to these issues in a forthcoming publication.

It is worth mentioning that the *super-D9-brane Lagrangian from $\mathcal{L}_{10}$ can be omitted form the action of the interacting system without loss of selfconsistency* (cf. [22]). Thus one may obtain a supersymmetric description of the coupled system of fundamental superstrings and lower dimensional super-Dp-branes ($p = 2k-1 < 9$), e.g. to the system of N coincident super-D3-branes which is of interest for applications to gauge theory [11, 12], as well as in the context of the Maldacena conjecture [49].

The only remaining trace of the D9-brane is the existence of a map (9.3) of the super-Dp-brane ($p < 9$) worldvolume into a 10-dimensional space whose coordinates are inert under a type II supersymmetry. Thus the system contains an auxiliary all-enveloping 9-brane ('9-brane dominance'). This means that we really do not need a space-time filling brane as a dynamical object and, thus, may be able to extend our approach to the $D = 10$ type IIA and $D = 11$ cases, where such dynamical branes are not known.

Another interesting direction for future study is to replace the action of the space-time filling brane by a counterpart of the group-manifold action for the corresponding supergravity theory (see [50]). Such an action also implies the map of a D-dimensional bosonic surface into a space with D-bosonic dimensions, as the space time filling brane does. Thus we can define an induced map of the worldvolumes of superstrings and lower branes into the D–dimensional bosonic surface involved in the group manifold action and construct the covariant action for the coupled system of intersecting superbranes *and supergravity.*

In this respect the problem to construct the counterpart of a group-manifold actions for the $D = 10$ *type II* supergravity [51] and duality-symmetric $D = 11$ supergravity [32] seems to be of particular interest.

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Appendix A. Properties of Lorentz harmonic variables

The Lorentz harmonic variables $u_{\underline{m}}^{\underline{a}}$, $v_{\underline{\mu}}^{\underline{\alpha}}$ parameterizing the coset

$$\frac{SO(1,9)}{SO(1,1)\otimes SO(8)}$$

which are used in the geometric action like (4.1) for $D=10$ superstring models.

Vector harmonics

In any number of space–time dimensions the Lorentz harmonic variables [45] which are appropriate to adapt the target space vielbein to the string world volume [14] are defined as $SO(1,D-1)$ group valued $D\times D$ matrix

$$u_{\underline{m}}^{\underline{a}} \equiv \left(u_{\underline{m}}^{0}, u_{\underline{m}}^{i}, u_{\underline{m}}^{9}\right) \equiv \left(\frac{u_{\underline{m}}^{++}+u_{\underline{m}}^{--}}{2}, u_{\underline{m}}^{i}, \frac{u_{\underline{m}}^{++}-u_{\underline{m}}^{--}}{2}\right) \quad \in \quad SO(1,D-1), \tag{A.1}$$

$$\Leftrightarrow \qquad u_{\underline{m}}^{\underline{a}} u^{\underline{b}\underline{m}} = \eta^{\underline{a}\underline{b}} \equiv diag(+1,-1,...,-1). \tag{A.2}$$

In the light-like notations

$$u_{\underline{m}}^{0} = \frac{u_{\underline{m}}^{++}+u_{\underline{m}}^{--}}{2}, \qquad u_{\underline{m}}^{9} = \frac{u_{\underline{m}}^{++}-u_{\underline{m}}^{--}}{2} \tag{A.3}$$

the flat Minkowski metric acquires the form

$$\Leftrightarrow \qquad u_{\underline{m}}^{\underline{a}} u^{\underline{b}\underline{m}} = \eta^{\underline{a}\underline{b}} \equiv \begin{pmatrix} 0 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & I_{8\times 8} \end{pmatrix}, \tag{A.4}$$

and the orthogonality conditions look like [45]

$$\Leftrightarrow \qquad u_{\underline{m}}^{++}u^{++\underline{m}} = 0, \qquad u_{\underline{m}}^{--}u^{--\underline{m}} = 0, \qquad u_{\underline{m}}^{++}u^{--\underline{m}} = 2,$$
$$u_{\underline{m}}^{i}u^{++\underline{m}} = 0, \qquad u_{\underline{m}}^{i}u^{++\underline{m}} = 0, \qquad u_{\underline{m}}^{i}u^{j\underline{m}} = -\delta^{ij}.$$

A2. Spinor Lorentz harmonics

For supersymmetric strings and branes we need to introduce the matrix $v_{\underline{\mu}}^{\underline{\alpha}}$ which takes its values in the double covering $Spin(1,D-1)$ of the Lorentz group $SO(1,D-1)$ and provides the (minimal) spinor representation of the pseudo-rotation whose vector representation is given by the vector harmonics u (spinor Lorentz harmonics [52, 46, 31]). The latter fact implies the invariance of the gamma–matrices with respect to the Lorentz group transformations described by u and v harmonics

$$v_{\underline{\mu}}^{\underline{\alpha}} \quad \in \quad Spin(1,D-1) \quad \Leftrightarrow \quad u_{\underline{m}}^{\underline{a}}\Gamma^{\underline{m}}_{\underline{\mu}\underline{\nu}} = v_{\underline{\mu}}^{\underline{\alpha}}\Gamma^{\underline{a}}_{\underline{\alpha}\underline{\beta}}v_{\underline{\nu}}^{\underline{\beta}}, \qquad u_{\underline{m}}^{\underline{a}}\Gamma_{\underline{a}}^{\underline{\alpha}\underline{\beta}} = v_{\underline{\mu}}^{\underline{\alpha}}\Gamma_{\underline{m}}^{\underline{\mu}\underline{\nu}}v_{\underline{\nu}}^{\underline{\beta}}. \tag{A.5}$$

In this paper we use the $D=10$ spinor Lorentz harmonic variables $v_{\underline{\mu}}^{\underline{\alpha}}$ parameterizing the coset $Spin(1,9)/[Spin(1,1)\times SO(8)]$ [31], which are adequate for the description of $D=10$

superstrings. The splitting (A.1) is reflected by the splitting of the 16×16 Lorentz harmonic variables into two 16×8 blocks

$$v_{\underline{\mu}}^{\ \underline{\alpha}} \equiv (v_{\underline{\mu}q}^{\ +}, v_{\underline{\mu}\dot{q}}^{\ -}), \qquad \in \qquad Spin(1,9)$$

$$v_{\underline{\alpha}}^{\ \underline{\mu}} \equiv (v_q^{-\underline{\mu}}, v_{\dot{q}}^{+\underline{\mu}}), \qquad \in \qquad Spin(1,9) \tag{A.6}$$

$$v_{\underline{\alpha}}^{\ \underline{\mu}} v_{\underline{\mu}}^{\ \underline{\beta}} = \delta_{\underline{\alpha}}^{\ \underline{\beta}}, \qquad v_{\underline{\mu}}^{\ \underline{\alpha}} v_{\underline{\alpha}}^{\ \underline{\nu}} = \delta_{\underline{\mu}}^{\ \underline{\nu}}, \qquad \Leftrightarrow$$

$$v_p^{-\underline{\mu}} v_{\underline{\mu}q}^{\ +} = \delta_{pq}, \qquad v_{\dot{p}}^{+\underline{\mu}} v_{\underline{\mu}\dot{q}}^{\ -} = \delta_{\dot{p}\dot{q}}, \qquad v_{\dot{q}}^{+\underline{\mu}} v_{\underline{\mu}q}^{\ +} = 0 = v_q^{-\underline{\mu}} v_{\underline{\mu}\dot{q}}^{\ -}, \tag{A.7}$$

$$\delta_{\underline{\mu}}^{\underline{\nu}} = v_q^{-\underline{\nu}} v_{\underline{\mu}q}^{\ +} + v_{\dot{q}}^{+\underline{\nu}} v_{\underline{\mu}q}^{\ -}.$$

To write in detail the relations (A.5) between spinor and vector Lorentz harmonics we need the explicit $SO(1,1) \times SO(8)$ invariant representation for the $D = 10$ Majorana–Weyl gamma matrices $\sigma^{\underline{a}}$

$$\sigma_{\underline{\alpha\beta}}^{\underline{0}} = \ diag(\delta_{qp}, \delta_{\dot{q}\dot{p}}) = \tilde{\sigma}^{\underline{0}\ \underline{\alpha\beta}}, \qquad \sigma_{\underline{\alpha\beta}}^{\underline{9}} = \ diag(\delta_{qp}, -\delta_{\dot{q}\dot{p}}) = -\tilde{\sigma}^{\underline{9}\ \underline{\alpha\beta}},$$

$$\sigma_{\underline{\alpha\beta}}^{i} = \begin{pmatrix} 0 & \gamma_{q\dot{p}}^{i} \\ \tilde{\gamma}_{\dot{q}p}^{i} & 0 \end{pmatrix} = -\tilde{\sigma}^{i\ \underline{\alpha\beta}}, \tag{A.8}$$

$$\sigma_{\underline{\alpha\beta}}^{++} \equiv (\sigma^{\underline{0}} + \sigma^{\underline{9}})_{\underline{\alpha\beta}} = \ diag(\ 2\delta_{qp},\ 0) = -(\tilde{\sigma}^{\underline{0}} - \tilde{\sigma}^{\underline{9}})^{\underline{\alpha\beta}} = \tilde{\sigma}^{--\ \underline{\alpha\beta}},$$

$$\sigma_{\underline{\alpha\beta}}^{--} \equiv (\sigma^{\underline{0}} - \Gamma^{\underline{9}})_{\underline{\alpha\beta}} = \ diag(\ 0,\ 2\delta_{\dot{q}\dot{p}}) = (\tilde{\sigma}^{\underline{0}} + \tilde{\sigma}^{\underline{9}})^{\underline{\alpha\beta}} = \tilde{\sigma}^{++\ \underline{\alpha\beta}},$$

where $\gamma_{q\dot{q}}^{i} = \tilde{\gamma}_{\dot{q}q}^{i}$ are 8×8 chiral gamma matrices of the $SO(8)$ group.

Substituting (A.8) we get from (A.5) [46, 31, 14]

$$\begin{aligned} u_{\underline{m}}^{++} \sigma_{\underline{\mu\nu}}^{\underline{m}} &= 2 v_{\underline{\mu}q}^{\ +} v_{\underline{\mu}q}^{\ +}, & u_{\underline{m}}^{++} \tilde{\sigma}^{\underline{m}\ \underline{\mu\nu}} &= 2 v_{\dot{q}}^{+\underline{\mu}} v_{\dot{q}}^{+\underline{\nu}}, \\ u_{\underline{m}}^{--} \sigma_{\underline{\mu\nu}}^{\underline{m}} &= 2 v_{\underline{\mu}\dot{q}}^{\ -} v_{\underline{\mu}\dot{q}}^{\ -}, & u_{\underline{m}}^{--} \tilde{\sigma}^{\underline{m}\ \underline{\mu\nu}} &= 2 v_q^{-\underline{\mu}} v_q^{-\underline{\nu}}, \\ u_{\underline{m}}^{i} \sigma_{\underline{\mu\nu}}^{\underline{m}} &= 2 v_{\{\underline{\mu}q}^{\ +} \gamma_{q\dot{q}}^{i} v_{\underline{\nu}\}\dot{q}}^{\ -}, & u_{\underline{m}}^{i} \tilde{\sigma}^{\underline{m}\ \underline{\mu\nu}} &= -2 v_q^{-\{\underline{\mu}} \gamma_{q\dot{q}}^{i} v_{\dot{q}}^{+\underline{\nu}\}}, \end{aligned} \tag{A.9}$$

$$u_{\underline{m}}^{i} \gamma_{q\dot{q}}^{i} = v_q^{+} \tilde{\sigma}_{\underline{m}} v_{\dot{q}}^{-} = -v_q^{-} \sigma_{\underline{m}} v_{\dot{q}}^{+},$$

$$u_{\underline{m}}^{++} \delta_{pq} = v_q^{+} \tilde{\sigma}_{\underline{m}} v_p^{+}, \qquad u_{\underline{m}}^{--} \delta_{\dot{p}\dot{q}} = v_{\dot{q}}^{-} \tilde{\sigma}_{\underline{m}} v_{\dot{p}}^{-}.$$

The differentials of the harmonic variables are calculated easily by taking into account the conditions (A.1), (A.6). For the vector harmonics this implies

$$du_{\underline{m}}^{\underline{a}} u^{\ \underline{b}\underline{m}} + u^{\ \underline{a}\underline{m}} du_{\underline{m}}^{\underline{b}} = 0,$$

whose solution is given by

$$du_{\underline{m}}^{\ \underline{a}} = u_{\underline{m}}^{\underline{b}} \Omega_{\underline{b}}^{\ \underline{a}}(d) \qquad \Leftrightarrow \qquad \begin{cases} du_{\underline{m}}^{++} = u_{\underline{m}}^{++} \omega + u_{\underline{m}}^{\ i} f^{++i}(d), \\ du_{\underline{m}}^{--} = -u_{\underline{m}}^{--} \omega + u_{\underline{m}}^{\ i} f^{--i}(d), \\ du_{\underline{m}}^{i} = -u_{\underline{m}}^{j} A^{ji} + \frac{1}{2} u_{\underline{m}}^{++} f^{--i}(d) + \frac{1}{2} u_{\underline{m}}^{--} f^{++i}(d), \end{cases} \tag{A.10}$$

where

$$\Omega_{\underline{b}}{}^{\underline{a}} \equiv u_{\underline{b}}^{\underline{m}} du_{\underline{m}}^{\underline{a}} = \begin{pmatrix} \omega & 0 & \frac{1}{\sqrt{2}} f^{--i}(d) \\ 0 & -\omega & \frac{1}{\sqrt{2}} f^{++i}(d) \\ \frac{1}{\sqrt{2}} f^{++i}(d) & \frac{1}{\sqrt{2}} f^{--i}(d) & A^{ji}(d) \end{pmatrix}, \qquad \Omega^{\underline{ab}} \equiv \eta^{\ \underline{ac}} \Omega_{\underline{c}}{}^{\underline{b}} = -\Omega^{\underline{ba}} \quad \text{(A.11)}$$

are $SO(1, D-1)$ Cartan forms. They can be decomposed into the $SO(1,1) \times SO(8)$ covariant forms

$$f^{++i} \equiv u_{\underline{m}}^{++} du^{\underline{m}i} \tag{A.12}$$

$$f^{--i} \equiv u_{\underline{m}}^{--} du^{\underline{m}i}, \tag{A.13}$$

parameterizing the coset $\frac{SO(1,9)}{SO(1,1)\times SO(8)}$, the $SO(1,1)$ spin connection

$$\omega \equiv \frac{1}{2} u_{\underline{m}}^{--} du^{\underline{m}\ ++}\ , \tag{A.14}$$

and $SO(8)$ connections (induced gauge fields)

$$A^{ij} \equiv u_{\underline{m}}^{i} du^{\underline{m}\ j}\ . \tag{A.15}$$

The Cartan forms (A.11) satisfy the Maurer-Cartan equation

$$d\Omega^{\underline{a}\ \underline{b}} - \Omega^{\underline{a}}{}_{\underline{c}} \wedge \Omega^{\underline{cb}} = 0 \tag{A.16}$$

which appears as integrability condition for Eq.(A.10). It has the form of a zero curvature condition. This reflects the fact that the $SO(1,9)$ connections defined by the Cartan forms (A.11) are trivial.

The Maurer–Cartan equation (A.16) splits naturally into

$$\mathcal{D}f^{++i} \equiv df^{++i} - f^{++i} \wedge \omega + f^{++j} \wedge A^{ji} = 0 \tag{A.17}$$

$$\mathcal{D}f^{--i} \equiv df^{--i} + f^{--i} \wedge \omega + f^{--j} \wedge A^{ji} = 0 \tag{A.18}$$

$$\mathcal{R} \equiv d\omega = \frac{1}{2} f^{--i} \wedge f^{++i} \tag{A.19}$$

$$R^{ij} \equiv dA^{ij} + A^{ik} \wedge A^{kj} = -f^{-[i} \wedge f^{+j]} \tag{A.20}$$

giving rise to the Peterson–Codazzi, Gauss and Ricci equations of classical Surface Theory (see [53]).

The differentials of the spinor harmonics can be expressed in terms of the same Cartan forms (A.12)–(A.15)

$$dv_{\underline{\mu}}^{\ \underline{\alpha}} = \frac{1}{4} \Omega^{\underline{ab}}\ v_{\underline{\mu}}^{\ \underline{\beta}}\ (\sigma_{\underline{ab}})_{\underline{\beta}}^{\ \underline{\alpha}}. \tag{A.21}$$

Using (A.8) we can specify (A.21) as (cf. [31])

$$v_p^{-\underline{\mu}} dv_{\underline{\mu}q}^{\ +} = \frac{1}{2}\delta_{pq}\omega - \frac{1}{4} A^{ij} \gamma^{ij}{}_{pq}, \qquad v_{\dot{p}}^{+\underline{\mu}} dv_{\underline{\mu}\dot{q}}^{\ -} = -\frac{1}{2}\delta_{\dot{p}\dot{q}}\omega - \frac{1}{4} A^{ij} \tilde{\gamma}^{ij}{}_{\dot{p}\dot{q}}, \tag{A.22}$$

$$v_{\dot{p}}^{+\underline{\mu}} dv_{\underline{\mu}q}^{\ +} = \frac{1}{2} f^{++i} \gamma^i_{q\dot{p}}, \qquad v_q^{-\underline{\mu}} dv_{\underline{\mu}\dot{p}}^{\ -} = \frac{1}{2} f^{--i} \gamma^i_{q\dot{p}} \tag{A.23}$$

Note that in $D=10$ the relations between vector (v–), c–spinor and s–spinor representations of the $SO(8)$ connections have the completely symmetric form

$$A_{pq} = \frac{1}{4} A^{ij} \gamma^{ij}{}_{pq}, \qquad A_{\dot{p}\dot{q}} = \frac{1}{4} A^{ij} \tilde{\gamma}^{ij}{}_{\dot{p}\dot{q}},$$

$$A^{ij} = \frac{1}{4} A_{pq} \gamma^{ij}{}_{pq} = \frac{1}{4} A_{\dot{p}\dot{q}} \tilde{\gamma}^{ij}{}_{\dot{p}\dot{q}},$$

This expresses the well known triality property of the $SO(8)$ group (see e.g. [20] and refs therein).

Appendix B. Linearized bosonic equations of type IIB superstring

Here we present the derivation of the linearized bosonic equations (4.27) of the superstring from the set of equations (4.19), (4.20).

In the gauge (4.24), (4.23) fermionic inputs disappear from Eq. (4.20). Moreover, in the linearized approximation we can replace $\hat{E}^{\pm\pm}$ by the closed form $d\xi^{\pm\pm}$ (holonomic basis for the space tangent to the worldsheet) and solve the linearized Peterson-Codazzi equations (A.17), (A.18)

$$df^{++i} = 0, \qquad df^{--i} = 0 \tag{B.1}$$

in terms of two $SO(8)$–vector densities $k^{++i}, k^{--i} = 0$ (infinitesimal parameters of the coset $SO(1,9)/[SO(1,1) \times SO(8)]$)

$$f^{++i} = 2dk^{++i}, \qquad f^{--i} = 2dk^{--i}. \tag{B.2}$$

Then the linearized form of the equations (4.19), (4.20) is

$$dX^i - \xi^{++} dk^{--i} - \xi^{--i} dk^{++i} = 0, \tag{B.3}$$

$$d\xi^{--i} \wedge dk^{++i} - d\xi^{++} \wedge dk^{--i} = 0. \tag{B.4}$$

Eq. (B.4) implies

$$\partial_{++} k^{++i} + \partial_{--} k^{--i} = 0,$$

while the integrability conditions for Eq. (B.3) are

$$d\xi^{--i} \wedge dk^{++i} + d\xi^{++} \wedge dk^{--i} = 0. \qquad \rightarrow \qquad \partial_{++} k^{++i} - \partial_{--} k^{--i} = 0 \tag{B.5}$$

Hence we have

$$\partial_{++} k^{++i} = \partial_{--} k^{--i} = 0. \tag{B.6}$$

Now, extracting, e.g. the component of (B.3) proportional to $d\xi^{++}$ and taking into account (B.6) one arrives at

$$\partial_{++} X^i = \xi^{++i} \partial_{++} k^{--i}. \tag{B.7}$$

The ∂_{--} derivative of Eq. (B.7) again together with Eq. (B.6) yields a relation which includes the X^i field only

$$\partial_{--} \partial_{++} X^i = \xi^{++i} \partial_{++} \partial_{--} k^{--i} = 0 \tag{B.8}$$

and is just the free equation (4.27).

Appendix C: The gauge field of D9-brane described by block–triangular spin-tensor h

Here we will present the nontrivial solution of the characteristic equation (3.20) for the spin–tensor h of the triangle form (8.19)

$$\hat{h}_{\underline{\beta}}{}^{\underline{\alpha}} \equiv \hat{v}_{\underline{\beta}}{}^{\underline{\nu}} \hat{h}_{\underline{\mu}}{}^{\underline{\nu}} \hat{h}_{\underline{\mu}}{}^{\underline{\alpha}} \equiv \begin{pmatrix} 0 & \hat{h}^{--}_{q\dot{p}} \\ \hat{h}^{++}_{\dot{q}p} & \tilde{\hat{h}}_{\dot{q}\dot{p}} \end{pmatrix} \qquad \in \quad Spin(1,9). \tag{C.1}$$

It corresponds to the $SO(1,9)$ valued matrix (cf. (3.21))

$$k_{\underline{a}}{}^{\underline{b}} \equiv u_{\underline{a}}{}^{m} k_{\underline{m}}{}^{\underline{n}} u_{\underline{n}}{}^{\underline{b}} \equiv \left(\frac{k^{++}_{\underline{a}} + k^{--}_{\underline{a}}}{2}, k^{i}_{\underline{a}}, \frac{k^{++}_{\underline{a}} - k^{--}_{\underline{a}}}{2} \right) \qquad \in \quad SO(1,9) \tag{C.2}$$

with the components

$$k^{++}_{\underline{a}} = \frac{1}{2} \delta^{--}_{\underline{a}} k^{++|++}, \tag{C.3}$$

$$k^{--}_{\underline{a}} = \delta^{++}_{\underline{a}} \frac{2}{k^{++|++}} + \delta^{--}_{\underline{a}} \frac{k^{++j} k^{++j}}{2k^{++|++}} - \delta^{i}_{\underline{a}} \frac{2k^{ij} k^{++j}}{k^{++|++}}, \tag{C.4}$$

$$k^{i}_{\underline{a}} = \frac{1}{2} \delta^{--}_{\underline{a}} k^{++i} - \delta^{j}_{\underline{a}} k^{ji}. \tag{C.5}$$

The matrix k^{ij}, entering (C.4), (C.5), takes its values in the $SO(8)$ group:

$$k^{ik} k^{jk} = \delta^{ij} \qquad \Leftrightarrow \qquad k^{ij} \in SO(8). \tag{C.6}$$

The nonvanishing 8×8 blocks of the 16×16 matrix h (C.1) are related with the independent components $k^{++|++}$, k^{++i}, $k^{ij} \in SO(8)$ of the matrix (C.2) by

$$h^{--}_{q\dot{s}} h^{--}_{p\dot{s}} = \delta_{qp} \frac{2}{k^{++|++}}, \tag{C.7}$$

$$h^{--}_{q\dot{s}} \tilde{h}_{\dot{q}\dot{s}} = -\gamma^{i}_{q\dot{q}} \frac{k^{ij} k^{++j}}{2k^{++|++}}, \tag{C.8}$$

$$h^{++}_{\dot{q}s} h^{++}_{\dot{p}s} = \delta_{\dot{q}\dot{p}} \frac{k^{++|++}}{2}, \tag{C.9}$$

$$\tilde{h}_{\dot{q}\dot{s}} \tilde{h}_{\dot{p}\dot{s}} = \delta_{\dot{q}\dot{p}} \frac{k^{++j} k^{++j}}{2k^{++|++}}, \tag{C.10}$$

$$h^{++}_{\dot{q}s} \gamma^{i}_{s\dot{s}} h^{--}_{q\dot{s}} = -\gamma^{j}_{q\dot{q}} k^{ji}, \tag{C.11}$$

$$2h^{++}_{(\dot{q}|s} \gamma^{i}_{s\dot{s}} \tilde{h}_{|\dot{s})\dot{s}} = \delta_{\dot{q}\dot{p}} k^{++i}. \tag{C.12}$$

These equations are produced by Eq. (3.20) in the frame related to the harmonics (4.3), (4.12) of the fundamental superstring.

The expression connecting the independent components $k^{++|++}$, k^{++i}, $k^{ij} \in SO(8)$ of the matrix (C.2) with the components of the antisymmetric tensor F (which becomes the field strength of the gauge field of the super–D9–brane on the mass–shell)

$$F_{\underline{ab}} \equiv u^{a}_{\underline{a}} F_{ab} u^{b}{}_{\underline{b}} = -F_{\underline{b}\,\underline{a}} = (F^{--|++}, F^{++i}, F^{--i}, F^{ij})$$

can be obtained from Eq. (3.21) in the frame related to the stringy harmonics

$$F^{--|++} = 2, \tag{C.13}$$

$$F^{++i} = -\frac{1}{2}k^{++|++}F^{--i}, \tag{C.14}$$

$$F^{--j}k^{ji} = F^{--i} \Leftrightarrow F^{--j}(\delta^{ji} - k^{ji}) = 0, \tag{C.15}$$

$$F^{--i}k^{++i} = 4, \tag{C.16}$$

$$F^{--i}k^{++j}k^{++j} = -4k^{ij}k^{++j} - 4F^{ij'}k^{j'j}k^{++j} \equiv -4(\delta^{ij} + F^{ij})4k^{jj'}k^{++j'}, \tag{C.17}$$

$$F^{ij'}(\delta^{j'j} - k^{j'j}) = -(\delta^{ij} + k^{ij}) + \frac{1}{2}F^{--i}k^{++j}. \tag{C.18}$$

In particular, the above results demonstrate that Eq. $h_{pq} = 0$ ((8.18) or (C.1)) indeed implies (8.20) (see (C.13)).

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